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Evolution of weak waves and central expansion waves in a non-ideal relaxing gas

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Abstract The main features of small amplitude waves generated by a planar piston-like surface in a planar flow of a vibrationally non-ideal relaxing gas are investigated. It is found that the analytical solution of the flow field for weak waves and central expansion wave propagation is influenced by the relaxation time and van der Waals excluded volume.

1. Introduction

A characteristic perturbation scheme for nonlinear hyperbolic waves has received great attention during the past decade; it has produced several new and interesting results, which may find numerous applications in the field of continuum mechanics. Using the same and some other related features to nonlinear waves, Rarity [1], Chou and Chu [2], Chu [3], Kumar et al. [4] and Singh et al. [5] have studied the nonlinear effects in their works. Wen-rui [6] used the perturbation method to study the weak shock and strong shock problems generated by the piston motion in weak gravitational field. Problems of propagation of weak planar and non-planar shock waves in uniform and non-uniform gases have been investigated using the perturbation scheme [7,8]. Xue [9] investigated the problem of non-planar dust-ion acoustic shock waves with transverse perturbation and deduced the Kadomtsev–Petviashvili Burgers equation that describes the dust-ion acoustic shock waves.

Assuming the electrical conductivity to be infinite and direction of the magnetic field to be orthogonal to the trajectories of gas particles, Singh et al. [10] used a systematic perturbation scheme to obtain the analytical solution of the flow field in non-uniform, radiative magnetogasdynamics. It is well known that in the ideal gas case, the compressive phase of the wave profile always steepens up into a shock wave [4]. Another interesting features of a relaxing gas lie in the observation that the far-field behavior of small amplitude motion is governed by Burger's equation, the solution of which exhibits the property that any convective steepening is always diffused by the diffusive nature of the relaxation [11]. However, the presence of relaxation offers an interesting situation in which the complete flow field is shock free [12]. Apart from all these, the system governing the motion of a relaxing gas possesses several novel features that make it worth of further study.

In this paper, we consider the one-dimensional planar motion of a non-ideal relaxing gas in a semi-infinite long tube, fitted with a piston at one end. The weak waves and expansive

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waves are generated by an impulsive start of a piston from rest with a nonzero but finite acceleration. The paper is organized as follows: after transforming the basic equations in characteristic coordinate and jump conditions across a shock of arbitrary strength, we devote Section 3 to the derivation of flow field using the characteristics perturbation scheme. In Section 4, assuming the piston acceleration is infinitely large, we determine the field variables near the central expansion wave. The last section consists of some final remarks and conclusions.

2. Characteristic transformation

The equations describing the one dimensional unsteady planar flow of non-ideal relaxing gases [11,13,14] are

$$\begin{aligned} &\rho_t + u\rho_x + \rho u_x = 0,\\ &u_t + uu_x + \frac{1}{\rho}p_x = 0,\\ &p_t + up_x + \rho a^2 u_x = -(\gamma-1)\rho Q,\\ &\sigma_t + u\sigma_x = Q, \end{aligned} \tag{1}$$

where $\rho \geqslant 0$, u, $p \geqslant 0$ and $\sigma \geqslant 0$ denote respectively the density, velocity, pressure and vibrational energy; the equilibrium speed of sound a is known function defined as $a = \left(\frac{\gamma p}{(1-b\rho)\rho}\right)^{1/2}$ where γ is the specific heat ratio lying in the region $1 < \gamma < 2$ for most gases and b is van der Waals excluded volume, which lies in the range $0.9 \times 10^{-3} \leqslant b \leqslant 1.1 \times 10^{-3}$ (SI unit is m^3 per unit mass). The independent variables t and x denote respectively time and space. It may be noted that the case $b = 0$ corresponds to the ideal relaxing gas. If not stated otherwise, an alphabet (variable) as a subscript indicates partial differentiation with respect to that variable. The quantity Q denotes the rate of change of vibrational energy, which is a function of p, ρ and σ, given by

$$Q = \frac{\bar{\sigma}(p,\rho) - \sigma}{\tau}, \tag{2}$$

where $\bar{\sigma} = \sigma_0 + c\{p(1-b\rho)/\rho - (1-b\rho_0)p_0/\rho_0\}$ is the equilibrium value of σ and the suffix 0 refers to the initial rest condition; the quantities τ and c are respectively the relaxation time and the ratio of vibrational specific heat to the specific gas constant. The situation $Q = 0$ corresponds to a physical process involving no relaxation; indeed, it includes the following cases

(i) The vibrational mode is inactive.
(ii) The vibrational mode follows the translational mode accordingly as the flow is frozen i.e. ($\sigma = const.$)
(iii) As $\tau \to \infty$.

The van der Waals equation of state is taken to be of the form

$$p(1 - b\rho) = \rho RT, \tag{3}$$

where R is the specific gas constant and T is the translational temperature. It is easy to see that the system (1) is hyperbolic with four families of characteristics, two of which $\frac{dx}{dt} = u \pm a$, represent waves propagating in the $\pm x$ directions with the effective speed $u \pm a$, and the remaining $\frac{dx}{dt} = u$ form a set of double characteristics and represent the particle paths or entropy waves. As discussed in [3], we consider the case where the boundary conditions are prescribed on the wave front and piston surface for observing the wave phenomena. It is convenient to use the characteristics of the governing system as the reference coordinate system. In order to study the weak wave and central expansion wave phenomena, we approach the wave front from the undisturbed medium and the boundary conditions $u = 0$, $\rho = \rho_0$, $p = p_0$ and $\sigma = \sigma_0$ on the wave front. If the piston path is given by $x = f(t)$, the piston speed satisfies the relation $u = f'(t)$, where the prime indicates the derivative with respect to the argument. We now introduce the characteristic variables ζ and η such that

$$\zeta_t + (u+a)\zeta_x = 0, \quad \eta_t + u\eta_x = 0. \tag{4}$$

If η is a particle tag so that η must be constant along the trajectory of the fluid particle $dx/dt = u$ in the $(x;t)$ plane. The particle and its path may be labeled by $\eta = t'$, when the characteristic wave front traverses a particle at time t' and if ζ is a wave tag so that ζ is constant along an outgoing characteristics $dx/dt = u + a$ in $(x;t)$ plane and will be labeled by $\zeta = t^*$, when an outgoing wave is generated at time t^* [3] (One may see Fig. 1). Hence using these particle path and outgoing characteristics, one may find the transformation relation between (x,t) and (ζ,η) as

$$x_\zeta = ut_\zeta, \quad x_\eta = (u+a)t_\eta. \tag{5}$$

The transformation from (x,t) plane to (ζ,η) plane is one to one iff the Jacobian $\mathcal{J} = -at_\zeta t_\eta$ is nonzero everywhere. Indeed, the solution in terms of the characteristic parameters will get up unbounded iff $t_\zeta = 0$. In terms of new characteristic coordinates ζ and η, the equations (1) can be written as

$$\begin{aligned} &(a\rho_\zeta - \rho u_\zeta)t_\eta + \rho u_\eta t_\zeta = 0, \quad (\rho a u_\zeta - p_\zeta)t_\eta + p_\eta t_\zeta = 0,\\ &(\rho a u_\zeta - p_\zeta)t_\eta - \rho a u_\eta t_\zeta = -(\gamma-1)t_\eta t_\zeta \rho Q, \quad \sigma_\zeta = -Qt_\zeta, \end{aligned} \tag{6}$$

with the following boundary conditions on the piston

$$t = \zeta, \quad x = f(\zeta), \quad u = f'(\zeta) \quad \text{at } \eta = 0. \tag{7}$$

The wave front is either a characteristic front or a shock wave and the boundary conditions depend on the wave front. If the wave front is a characteristic front, then all the flow variables are continuous across it, that is

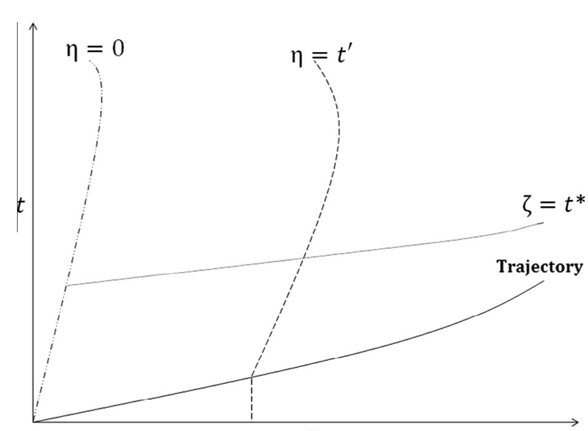

Figure 1 Characteristic transformation of physical plane to (ζ,η) coordinates.