Ain Shams Engineering Journal (2015) 6, 355–362

Ain Shams University

Ain Shams Engineering Journal

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ENGINEERING PHYSICS AND MATHEMATICS

Flow of a Jeffrey fluid between torsionally oscillating disks

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Received 30 June 2014; revised 2 September 2014; accepted 9 September 2014
Available online 7 October 2014

KEYWORDS

Jeffrey fluid;
Torsionary oscillating disks;
Frequency

Abstract In this paper, the flow of Jeffrey fluid between two torsionally oscillating disks is studied. This problem is solved in two cases. The first case is one disk oscillating and the other is at rest and the second case is two disks are oscillating with same frequency and speed but with phase difference of 180°. We found that the radial–axial flow has a mean steady component and a fluctuating component of frequency twice that of the oscillating disk. When the Jeffrey parameter λ tends to zero, the results coincide with the corresponding Newtonian case obtained by Rosenblat.

1. Introduction

The study of torsional vibrations of break disks is very important especially in applications where high power transmission and high speed are present. The model of flow between torsional oscillating disks may be observed in the turbine–coupling–generator rotor system and frictionless bearings. The torsional oscillation of a plate in Newtonian fluids has been discussed by Rosenblat [1]. He obtained the solution by expanding velocity components and the pressure in powers of the amplitude of oscillation of the plate and showed that the solution is highly convergent within the boundary layer. He has also discussed the case when the fluid is confined between two torsionally oscillating plates [1]. Similar problems in Reiner–Rivlin fluids were discussed by Srivastava [2,3]. In 1959, Rosenblat examined the flow between torsionally oscillating disks in the two cases: (i) one disk oscillating and the other at rest and (ii) both disks oscillating with the same frequency and speed, but with a phase difference of 180°. He developed and investigated the transverse and radial–axial flows for both small and large Reynold numbers. The theoretical analysis has been extended by Rajeswari [4] for Reiner–Rivlin fluids. She found that the radial–axial flow has a mean steady component and a fluctuating component of frequency twice that of the oscillating disk, a result similar to that for the Newtonian case obtained by Rosenblat. Bhatnagar and Rajeswari [4] and Srivastava have studied the same problem for a special case of the Rivlin–Ericksen second order fluid. Frater [5] has discussed only the first case of oldroyd fluid. Bhatnagar and Rajeswari have found that a reversal of the direction of the steady secondary flow is a characteristic feature of the Rivlin–Ericksen fluid and pointed out that it is always possible to find a value of the Reynolds number above which the flow is in reversed direction. The flow of an

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Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2014.09.004
2090-4479

Nomenclature

u	radial velocity component	υ	kinematic viscosity
v	transverse velocity component	R	Reynolds number
w	axial velocity component	n	frequency
p	pressure	Ω	angular speed
ρ	density	$\frac{\Omega}{n}$	amplitude
μ	dynamic viscosity		
λ	Jeffrey parameter		

incompressible visco-elastic Maxwell fluid between two parallel infinite disks executing small torsional oscillations in their own plane is discussed by Verma [6]. He found that with the increase of relaxation time parameter the elastic effects dominate in the region away from the oscillating disk and for small relaxation time, viscous effects permeate the entire flow.

Verma et al. [7] studied the flow induced in a viscous incompressible fluid from small torsional oscillations of an impermeable infinite disk bounded coaxially by another stationary naturally permeable infinite disk. He found that the steady radial velocity increases in magnitude with an increase of Reynolds number. Raghupathi Rao et al. [8] investigated the flow of a viscous fluid confined between two torsionally oscillating disks oscillating with the same frequency, but rotating with different, angular speeds about axes normal to the disks but not coincident. Sharma et al., studied the flow of an incompressible second-order fluid due to torsional oscillations of two infinite disks. They found that the effect of second order forces increases the amplitude of the oscillation of the axial velocity. The unsteady MHD flow of an incompressible viscous electrically conducting fluid contained between two torsionally oscillating eccentric disks has been investigated by Ragupathi Rao. Torsional oscillation of an infinite disk in a viscous liquid bounded by a porous medium fully saturated with the liquid was discussed by Srivastava. He found that the depth of penetration of the flow in the porous medium is proportional to the square root of the permeability of the medium.

Srivastava et al. discussed the flow due to torsional oscillations of infinite disks at a small distance from the unbounded porous medium when the entire space between the disks and the porous medium is filled with a second grade fluid. The problem of the flow of an incompressible non-Newtonian second order fluid between two enclosed torsionally oscillating disks has been discussed by Singh et al.. The effects of transducer compliance on transient stress measurements in torsional flow of a viscoelastic fluid are investigated by Dutcher et al. [9]. Pawan kumar Sharma et al. [10] investigated the unsteady laminar flow of an incompressible viscous electrically conducting fluid in a porous medium fully saturated with the liquid and bounded by torsionally oscillating disk in the presence of a transverse magnetic field.

Nadeem et al. [11] studied the effect of Jeffrey fluid with variable viscosity in the form of a well known Reynolds model of viscosity in an asymmetric channel. Akbar et al. [12] discussed a non-Newtonian fluid model for a blood flow through a tapered artery with a stenosis by assuming blood as Jeffrey fluid. Nadeem et al. [13] discussed the closed form analytical and numerical solutions of the peristaltic flow of a Jeffrey fluid in an inclined tube with different viscosities and with different wave shapes. Non-Newtonian fluid model for blood flow through a tapered artery with a stenosis and variable viscosity by modeling blood as Jeffrey fluid has been studied by Akbar et al. [14]. The effect of temperature-dependent viscosity on the Peristaltic flow of Jeffrey fluid through the gap between two co-axial horizontal tubes was analyzed by Akbar et al. [15]. Akbar et al. [16] studied a non-Newtonian fluid model for blood flow through a tapered artery with a stenosis by assuming blood as Jeffrey fluid. Hayat et al. [17] examined the flow of an incompressible Jeffrey fluid over a stretching surface. Hayat et al. [18] described the mixed convection stagnation point flow and heat transfer of a Jeffrey fluid toward a stretching surface. The boundary layer stretched flow of a Jeffrey fluid subject to the convective boundary conditions was investigated by Hayat et al. [19].

In this chapter, the flow of Jeffrey fluid between two torsionally oscillating disks is studied. This problem is solved in two cases. The first case is one disk oscillating and the other is at rest and the second case is two disks are oscillating with same frequency and speed but with phase difference of 180°. We found that the radial–axial flow has a mean steady component and a fluctuating component of frequency twice that of the oscillating disk. When the Jeffrey parameter λ tends to zero, the results coincide with the corresponding Newtonian case obtained by Rosenblat.

2. Mathematical formulation

We consider a body of a Jeffrey fluid bounded by two Infinite parallel plane disks which are represented by the plane $z = 0$ and $z = d$ in a cylindrical polar co-ordinate system. The disks perform torsional oscillations about the axis $r = 0$. If u, v and w be respectively the radial, transverse and axial velocity components, p be the pressure, ρ be the density, μ is the dynamic viscosity, λ is the ratio of relaxation to retardation time and v is the kinematic viscosity.

The constitutive equations for Jeffrey fluid (Vajravelu et al., [20]) are

$$T = -PI + S$$

$$S = \frac{\mu}{1+\lambda_1}[\dot{\gamma} + \lambda_2\ddot{\gamma}]$$

where P is pressure, S is extra stress tensor, T is the stress, I is identity tensor, μ is dynamic viscosity, λ_1 is the ratio of relaxation time and retardation times, λ_2 is the retardation time, γ is rate of strain tensor and the dots over the quantities denote differentiation. The quantities $\dot{\gamma}$ and $\ddot{\gamma}$ are defined by