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Forces between arrays of permanent magnets of basic geometric shapes

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ABSTRACT

We provide formulas for evaluating the magnetic force between two permanent magnet arrays, regularly spaced over a square lattice. We focus on three basic shapes of magnets constituting the arrays: cylinder, sphere and rectangular prism. When the lattice parameter is large, the expressions can be used to calculate the force between two single magnets in a computationally efficient way. The calculations are validated experimentally by measuring the attraction force between two single permanent magnets, where we demonstrate a fair agreement within about 15%.

1. Introduction

With growing availability of high-energy magnets of various types, forms and functionalities, the range of applications employing them is constantly expanding: magnetic separation [1], micro-robotic [2,3], magnetic force microscopy (MFM) [4], controlled motion of magnetic micro-particles in human body [5], energy harvesting [6], etc. Such applications are fundamentally based upon the magnetostatic interaction energy (MIE) between permanently magnetized bodies of various shapes arranged in certain geometries. From the knowledge of the MIE as a function of the relevant variables involved, mainly shape and relative displacement, one can derive acting forces and torques [7].

Several authors expressed analytically or semi-analytically the force acting between permanent magnets of basic shapes such as the interaction cylinder–cylinder [8–10], prism–prism [9,11] or tube–tube [12]. Interactions between magnet arrays and their images formed within a nearby superconducting body were also addressed by A.M. Campbell in [13]. However, magnets or arrays of magnets where bodies of different shapes interact are equally, if not more, important. For example, in the perspective of controlling the motion of spherical nanoparticles with either a cylindrical permanent magnet or a solenoid, the relevant interaction would be cylinder–sphere; many other combinations and permutations are possible, and several of those are of current interest in various types of devices.

Our formulas for the force between two arrays of permanent magnets of various shapes are based on the work of Beleggia and De Graef [14], combined with our recent paper on the interactions between arrays of magnets [15]. What distinguishes this manuscript from our previous work is the focus on giving manageable and useful analytical expression in terms of Fourier series that can be used to estimate reliably the forces in play between magnets of different shapes. In fact, in principle, the shape amplitude formalism we introduced to describe fields and energies of magnetized bodies [14] is capable of handling bodies of different shapes, although the MIE landscape for the general case is expressed in integral form. Here, by considering arrays, and, if needed, their limit for infinite lattice spacing, we translate those remaining integrals into Fourier series that can be implemented straightforwardly and efficiently in any calculator. We then give explicit examples of force calculations for the interactions between objects of the following basic shapes: cylinder–cylinder, cylinder–sphere, cylinder–prism, sphere–sphere, sphere–prism, prism–prism. We finally conclude by comparing the calculated forces with the results of measurements on FeNdB permanent magnets.

2. Theoretical framework

In [15], we determined the MIE and the magnetic force between two tightly packed arrays of cylindrical, axially magnetized permanent magnets with alternating orientation of magnetization within the arrays. A scheme of the arrays is shown in Fig. 1a: one of the arrays is composed of four magnets, whereas the other is infinite. We assume that the magnets in either set do not

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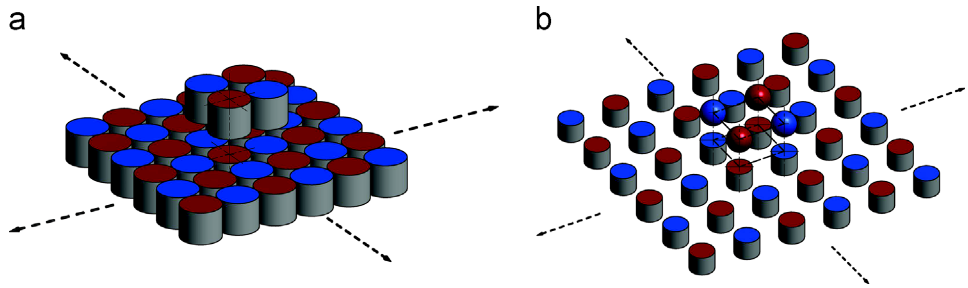

Fig. 1. Arrays of interacting permanent magnets. (a) The upper {bottom} set is formed by four {an infinite array of} tightly packed cylindrical magnets. All magnets are magnetized axially, and the magnetization orientation is denoted by blue (up) and red color (down). (b) Two sets of permanent magnets: an infinite array of cylindrical magnets forms the bottom set, while four spheres form the upper set. The distance between two closest magnets (centre-to-centre distance) in either set is denoted as $2R$, $R>r_c$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

change relative positions, and they can only move as a whole. For convenience, we review the force formula obtained in [15]:

$$F_z^{4/\infty}(\zeta,\tau) = -8\pi F_0 \sum_{i,j>0}^{odd} \frac{J_1^2(\rho_{ij})}{\rho_{ij}^2} \sinh^2(\tau\rho_{ij}) \exp(-\zeta\rho_{ij}) \tag{1}$$

where

$$\rho_{ij} = \frac{\pi}{2}\sqrt{i^2+j^2} \tag{2}$$

$F_0 = 4\pi\mu_0 M^2 r_c{}^2$ is the force constant, M is the saturation magnetization of the material, r_c is the cylinder's radius, τ is the cylinder's aspect ratio ($\tau = r_c/d$, where d is the cylinder's half-height), $\zeta = Z/r_c$ is the reduced axial distance, Z is the axial distance between the two cylinder sets ($Z \geq 2d$,) and $J_1(x)$ is a Bessel function of the first kind [16]. The symbol $\sum_{i,j>0}^{odd}$ indicates summation over all indexes i, j that are odd positive integers.

While for now we only treat the case where each magnet of the upper set has a common axis with a respective magnet of the bottom set, since, in general, it represents an equilibrium configuration versus lateral displacement, it would not be too difficult to generalize the description to a scenario where magnets are offset laterally, and this extension is subject of current efforts.

The magnet arrangement shown in Fig. 1a is representative of magnetic fasteners, where achieving the largest possible contact force (i.e. the force necessary to detach the magnet sets from $\zeta = 2\tau$) is the primary goal. In order to assess the force, in Ref. [15] we defined a force ratio $f(\tau)$ as follows

$$f(\tau) = \left| \frac{F_z^{4/\infty}(2\tau,\tau)}{4F_z^{1/1}(2\tau,\tau)} \right| \tag{3}$$

representing a measure of the array contact force enhancement with respect to interacting single bodies. The symbol $F_z^{1/1}$ indicates axial force between two aligned cylinders.

For the arrangement of Fig. 1a the ratio $f(\tau)$ is greater than 1 (see Fig. 2 in [15]). In this study, we are particularly interested in arrangements for which $f(\tau)$ is close to 1, that implies weak interactions between laterally displaced magnets in different arrays. Clearly, these "lateral interactions" decrease as the lattice constant of the arrays (as shown in Fig. 1b) increases. In the limiting case $R \to \infty$, where R is the half center-to-center distance between neighboring magnets within one set, we have $f(\tau) = 1$ and $F_z^{4/\infty} = 4F_z^{1/1}$.

As Fig. 1b suggests, the two sets (top or bottom) of magnets in the $(4/\infty)$ arrangements may be composed of differently shaped elements. The magnet centers within either set are coplanar and Z is the distance between the two central planes (where the magnets' centers lie). In the case of Fig. 1b, distance Z at contact equals $r_s + d$, where r_s and d are the sphere radius and the cylinder's half-height, respectively.

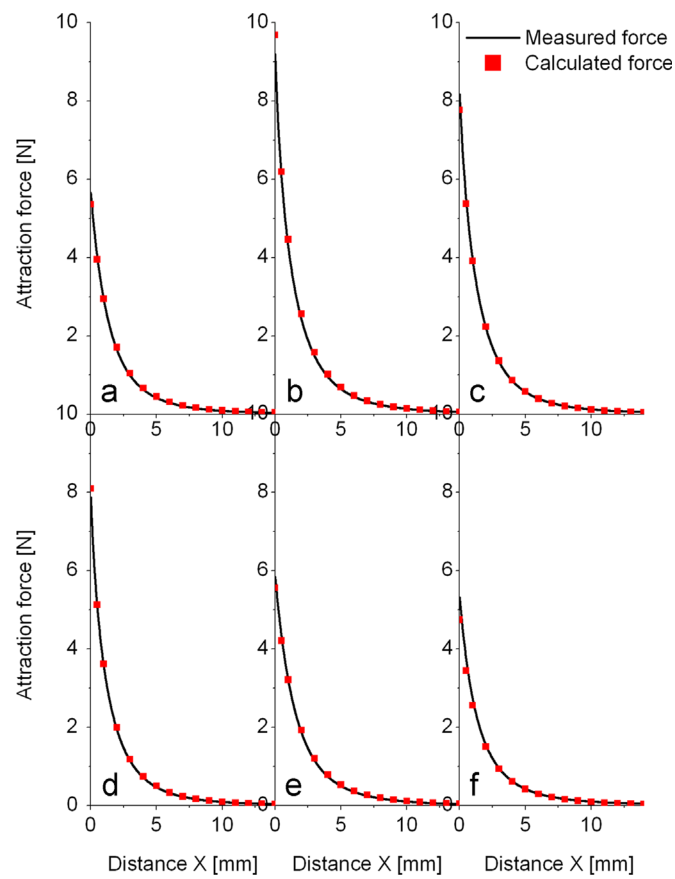

Fig. 2. Diagrams of the attraction force versus gap distance X for interactions (a) cylinder–sphere, (b) cube–cube, (c) cylinder–cube, (d) cylinder–cylinder, (e) cube–sphere, (f) sphere–sphere. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Our previous paper [15] deals with the MIE of two cylindrical arrays. Since we now deal with arrays of objects that may consist of various shapes, it is helpful to review briefly the general expression for the MIE. Within the Fourier-space framework described in [10,14], we can express the MIE of two uniformly magnetized objects A and B as

$$E_i(\boldsymbol{\rho}) = \frac{\mu_0}{8\pi^3} \mathrm{Re} \int \frac{d\mathbf{k}}{k^2} [\mathbf{M}_A(\mathbf{k}) \cdot \mathbf{k}][\mathbf{M}_B^*(\mathbf{k}) \cdot \mathbf{k}] e^{i\mathbf{k}\cdot\boldsymbol{\rho}} \tag{4}$$

where $\boldsymbol{\rho}$ is a displacement vector connecting the mass centers of bodies A and B, and $\mathbf{M}_{A,B}(\mathbf{k})$ are the Fourier transforms of the two magnetizations $\mathbf{M}_{A,B}(\mathbf{r})$, which include the shape information. The star above $\mathbf{M}$ denotes complex conjugation.