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Magnetic force on the walls of a square coaxial transmission line

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ABSTRACT

There is magnetic force acting on the wall of a square coaxial transmission line when TEM wave propagates in it. The self-inductance can be determined based on a conformal transformation for the field region, which demonstrates that the inductance only depends on the side length ratio of the two walls instead of the length of the walls. The magnetic force on each side of the walls is then calculated by employing the principle of virtual work. It is proved that the magnetic force has the same magnitude and opposite direction to the electric force. And then the electromagnetic force on the wall is zero. The general meaning of this conclusion for all kinds of coaxial transmission line is proposed.

1. Introduction

The coaxial line is an important type of transmission line used extensively in microwave engineering [1,2]. The electric force on the walls of some typical coaxial lines has been studied [3,4]. However, when TEM wave travels in a coaxial line, there is magnetic force acting on the walls [5]. The aim of this article is focused on determining the magnetic force on the walls of a square coaxial transmission line. The conformal mapping method is employed to calculate the self-inductance as the function of the side length ratio of two walls. Then magnetic energy is obtained. Using the principle of virtual work, the magnetic forces exerted on the walls of a square coaxial line are achieved with elliptic functions and integrals. Based on the result, the electric force on the wall is also studied. It is proved that magnetic force has the same magnitude and opposite direction to the electric force. So the electromagnetic force on wall is zero when TEM wave propagates in the coaxial line. This conclusion has the general meaning for all kinds of coaxial transmission lines.

2. Field region transformation

TEM wave travels in a square coaxial line. Fig. 1 illustrates the cross section of the line. Let l_1 and l_2 denote the side length of the inner and outer square wall respectively. When the longitudinal length of the line is sufficiently greater than l_2, the field $\boldsymbol{H}$ and $\boldsymbol{E}$ in this problem may be regarded as 2-D static field in the complex s-plane where the cross section is [6]. Suppose the current on the wall is I and the traverse voltage between two walls in the cross section is U, we may easily determine them from telegraph equations below [7]

$$\frac{\partial^2 I}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 I}{\partial t^2} = 0; \quad \frac{\partial^2 U}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 U}{\partial t^2} = 0 \tag{1}$$

Due to the symmetry, the field region $opqro$ in the s-plane is only considered. That region can be mapped onto the upper t-plane of Fig. 2 by the Schwarz–Crystoffel transformation [8]

$$\frac{ds}{dt} = \frac{1}{2k} t^{-3/4} (t-1)^{-1/2} \left(t - \frac{1}{k^2}\right)^{-1/2} \tag{2}$$

Introducing a transformation

$$t = u^2 \tag{3}$$

we map the field region in the upper t-plane to the first quadrant of u-plane depicted in Fig. 3.

Eq. (2) then becomes

$$s = \int_0^u \frac{du}{\sqrt{u(1-u^2)(1-k^2 u^2)}} \tag{4}$$

which is a super elliptic integral [9]. Integrating Eq. (4) gives

$$s = \frac{1}{\sqrt{2(1+k)}} \left[sn^{-1}(\xi, \lambda) + sn^{-1}(\xi, \lambda') \right] \tag{5}$$

where $sn^{-1}(\xi, \lambda)$ is an inverse elliptic function. The variable ξ is

$$\xi = \sqrt{\frac{2(1+k)u}{(1+u)(1+ku)}} \tag{6}$$

and the complementary moduli λ and λ' satisfy

$$\frac{K(\lambda)}{K(\lambda')} = \frac{1+\tau}{1-\tau}; \quad \lambda^2 + \lambda'^2 = 1 \tag{7}$$

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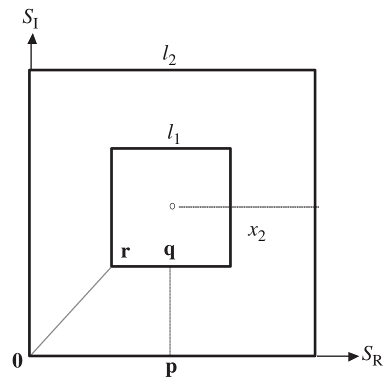

Fig. 1. The cross section of a square coaxial line in s-plane.

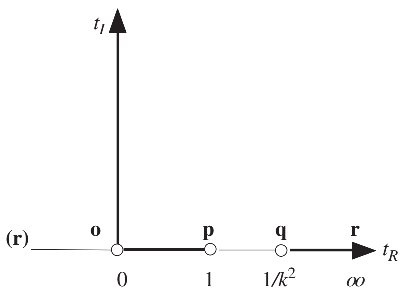

Fig. 2. The t-plane.

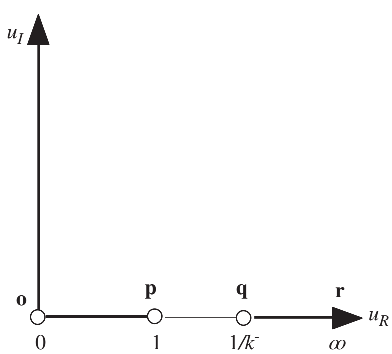

Fig. 3. The u-plane.

where $K(\lambda)$ is the complete elliptic integration of the first kind and τ is the ratio of the two side length

$$\tau = \frac{l_1}{l_2} \tag{8}$$

Finally, employing the following transformation

$$u = sn\vartheta \tag{9}$$

the field region in the first quadrant of the u-plane is mapped into the rectangle in the ϑ-plane of Fig. 4 [10]. Two side lengths of the rectangle are $K(k)$ and $K(k')$. The complementary moduli k and k' satisfy

$$k = \left(\frac{\lambda - \lambda'}{\lambda + \lambda'}\right)^2; \quad k^2 + k'^2 = 1 \tag{10}$$

3. Self-inductance

In the ϑ-plane the field is bounded by the rectangle and uniform. Notice the magnetic field is along the ϑ_R-direction, so the self-inductance of the square coaxial line per unit longitudinal length can be computed directly as [11]

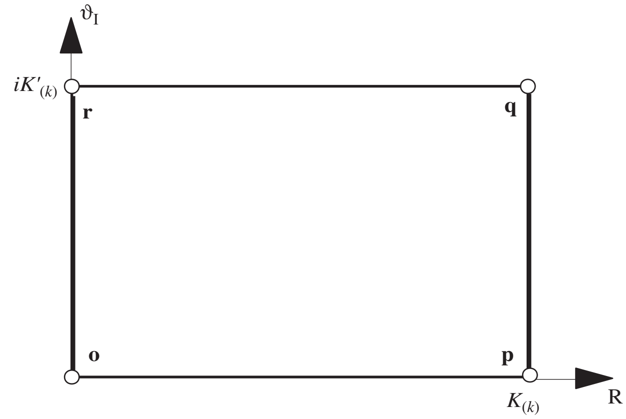

Fig. 4. The ϑ-plane.

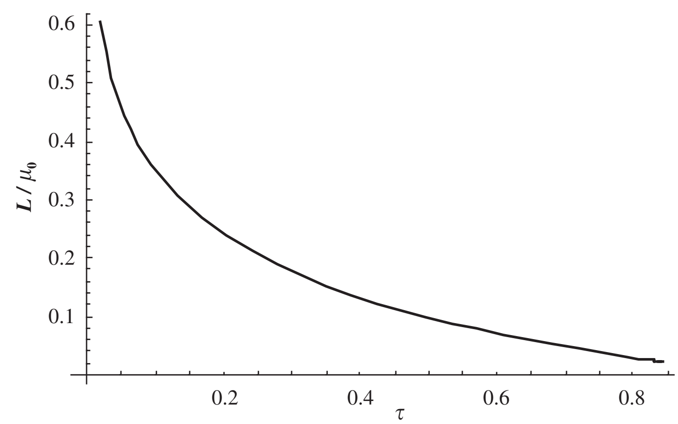

Fig. 5. The function curve of L/μ_0 versus τ.

$$L = \mu_0 \frac{K(k')}{8K(k)} \tag{11}$$

Eqs. (11), (10) and (7) demonstrate that the self-inductance only depends on the side length ratio of the two walls rather than the length of the walls. The function curve of the normalized self-inductance L/μ_0 versus the side length ratio τ is plotted in Fig. 5.

Hence, along the square coaxial line the linear density of magnetic energy is

$$w_m = \frac{1}{2} L I^2 \tag{12}$$

4. Magnetic force on the walls

Referring to Fig. 1, the distance between the center to the outer wall is

$$x_2 = l_2/2 \tag{13}$$

Regard x_2 as a generalized coordinate. Along the square coaxial line the linear density of magnetic force on each side of the outer wall can be determined by using the principle of virtual work [12]

$$f_2 = \frac{1}{4}\frac{\partial W_m}{\partial x_2}\Big|_{I=c} = \frac{1}{4}\frac{dl_2}{dx_2}\frac{\partial W_m}{\partial l_2}\Big|_{I=c} = \frac{1}{2}\frac{\partial W_m}{\partial l_2}_{\;I=c} \tag{14}$$