Alexandria Engineering Journal (2016) 55, 1305–1319

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Alexandria Engineering Journal

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ORIGINAL ARTICLE

# Finite element simulation of magnetohydrodynamic convective nanofluid slip flow in porous media with nonlinear radiation

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Received 24 January 2016; revised 27 February 2016; accepted 12 April 2016
Available online 30 April 2016

**KEYWORDS**

Darcy porous medium;
Finite element;
MHD slip flow;
Nonlinear radiation;
Zero mass flux

**Abstract** A numerical investigation of two dimensional steady state laminar boundary layer flow of a viscous electrically-conducting nanofluid in the vicinity of a stretching/shrinking porous flat plate located in a Darcian porous medium is performed. The nonlinear Rosseland radiation effect is taken into account. Velocity slip and thermal slip at the boundary as well as the newly developed zero mass flux boundary conditions are also implemented to achieve physically applicable results. The governing transport equations are reduced to a system of nonlinear ordinary differential equations using appropriate similarity transformations and these are then solved numerically using a variational finite element method (**FEM**). The influence of the governing parameters (Darcy number, magnetic field, velocity and thermal slip, temperature ratio, transpiration, Brownian motion, thermophoresis, Lewis number and Reynolds number) on the dimensionless velocity, temperature, nanoparticle volume fraction as well as the skin friction, the heat transfer rates and the mass transfer rates are examined and illustrated in detail. The FEM code is validated with earlier studies for non-magnetic non-slip flow demonstrating close correlation. The present study is relevant to high-temperature nano-materials processing operations.


## 1. Introduction

A suspension of nanometer-sized particles/fibers in a base liquid changes the carrier fluid properties (viscosity, density, thermal conductivity, mass diffusivity) and is regarded as a "nanofluid". Water, organic liquids, kerosene, lubricants, bio-fluids, and polymeric solution are normally used as base fluids. Nanoparticles are made from various materials such

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.04.021
1110-0168 

## Nomenclature

| | |
|---|---|
| $a$ | slip parameter (–) |
| $b$ | thermal slip parameter (–) |
| $B_0$ | constant magnetic field (telsa) |
| $C$ | dimensional concentration |
| $c_p$ | specific heat at constant pressure (J/kg K) |
| $C_{f\bar{x}}$ | skin friction factor (–) |
| $Da$ | Darcy number (–) |
| $Da_{\bar{x}}$ | local Darcy number (–) |
| $D_B$ | Brownian diffusion coefficient ($m^2$/s) |
| $D_T$ | thermophoretic diffusion coefficient ($m^2$/s) |
| $D_1$ | thermal slip factor (1/s) |
| $f(\eta)$ | dimensionless stream function (–) |
| $f_w$ | suction/injection parameter (–) |
| $K_p$ | permeability of the porous medium ($m^2$) |
| $k$ | thermal conductivity ($m^2$/s) |
| $k_1$ | Rosseland mean absorption coefficient (1/m) |
| $L$ | characteristic length of the plate (m) |
| $Le$ | Lewis number (–) |
| $M$ | magnetic field parameter (–) |
| $N_1(\bar{x})$ | variable velocity slip factor (s/m) |
| $Nb$ | Brownian motion parameter (–) |
| $Nt$ | thermophoresis parameter (–) |
| $Nu_{\bar{x}}$ | heat transfer rate |
| $Pr$ | Prandtl number (–) |
| $p$ | pressure (Pa) |
| $q_m$ | wall mass flux (kg/s $m^2$) |
| $q_w$ | wall heat flux (W/$m^2$) |
| $R$ | conduction–radiation parameter (–) |
| $Re$ | Reynolds number (–) |
| $Re_{\bar{x}}$ | local Reynolds number (–) |
| $Sh_{\bar{x}}$ | local Sherwood number (–) |
| $T$ | dimensional temperature (K) |
| $\bar{u}$, $\bar{v}$ | velocity components along the axes (m/s) |
| $\bar{u}_w$ | velocity of the plate (m/s) |
| $\bar{u}_e$ | external velocity (m/s) |
| $U_\infty$ | reference velocity (m/s) |
| $\bar{x}, \bar{y}$ | Cartesian coordinates aligned along and normal to the plate (m) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity ($m^2$/s) |
| $\alpha_i$ | real numbers (–) |
| $\beta$ | pressure gradient parameter (–) |
| $\tau$ | ratio of effective heat capacity of the nanoparticle material to the heat capacity of the fluid (–) |
| $\sigma$ | electric conductivity |
| $\sigma_1$ | Stefan–Boltzmann constant |
| $\theta(\eta)$ | dimensionless temperature (–) |
| $\theta_w$ | wall temperature excess ratio parameter (–) |
| $\phi(\eta)$ | dimensionless concentration (nanoparticle volume fraction) (–) |
| $\eta$ | similarity variable (–) |
| $\mu$ | dynamic viscosity of the fluid (Ns/$m^2$) |
| $\nu$ | kinematic viscosity of the fluid ($m^2$/s) |
| $\rho_f$ | fluid density (kg/$m^3$) |
| $\psi$ | stream function (–) |

*Subscripts*

| | |
|---|---|
| $w$ | condition at the wall |
| $\infty$ | ambient condition |

as oxide ceramics, metal nitride, carbide ceramics, metals and various forms of carbons [1]. Nanofluids have been proven to have diverse engineering and biomedical applications in both porous and purely fluid systems. Representative examples include the following: advanced nuclear systems, fuel cells and drug delivery. Micro-electro-mechanical systems (MEMS), nano-electro-mechanical systems (NEMS) and indeed many thermo-technical devices produce a huge amount of heat, which affects the normal activities of the devices and reduces the longevity. Hence, an efficient cooling (heating) system is required in designing such devices. It is known that heat transfer capacity of fluid can be improved by changing (i) flow geometry, (ii) boundary conditions, (iii) enhancing thermal conductivity, and (iv) using porous media [2]. Choi [3] has shown that nanoparticles can be used to enhance thermal conductivity of the base fluid. Transport phenomena associated with the flow of nanofluids have received the attention of investigators due to their diverse applications where heat and/or mass transfer play a vital role. The combination of a porous medium and nanofluid can significantly improve the effective heat transfer characteristics [4]. Therefore deployment of porous media in thermal engineering systems has been the subject of extensive activity in engineering sciences. Porous media arise in an astonishing range of technical and environmental applications including filtration, thermal insulation (fabrics and building materials), groundwater hydrology, oil reservoir formations, combustion, biomechanics, food stuffs and many types of heat exchanger [4]. In power station systems, cooling/heating is essential (e.g. in turbine blades), heat dissipation in electronic equipment and combustion systems (burners) to maintain efficient operation of the systems. The mixing of the low and high energy fluids which occur in these applications significantly affects the performance of these devices [5]. One of the ways to boost heat transfer (and cool boundaries) is to employ a porous medium saturated with nanofluid. Fundamental and detailed expositions of engineering applications of porous media such as solar energy collectors, heat exchangers, geothermics and heat pipes are documented in a number of monographs [6–9]. Nanofluids can and have been successfully utilized in many technologies, especially those that involve significant enhancement of heat transfer, and an excellent elaboration in this regard is Yu and Xie [10] (and references contained therein). For example, nanofluids can *increase the cooling rates* of heavy-duty engines by increasing the efficiency and reducing the complexity of thermal management systems and are a sustainable and relatively inexpensive mechanism for doing so [4]. So far, two popular mathematical formulations for nanofluid transport