Alexandria Engineering Journal (2016) 55, 69–75

HOSTED BY

ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

Entropy generation analysis for variable viscous couple stress fluid flow through a channel with non-uniform wall temperature

J.A. Falade [a], S.O. Adesanya [b,*], J.C. Ukaegbu [b], M.O. Osinowo [a]

[a] *Department of Physical Sciences, Redeemer's University, Ede, Nigeria*
[b] *Department of Mathematical Sciences, Redeemer's University, Ede, Nigeria*

Received 10 December 2015; revised 5 January 2016; accepted 19 January 2016
Available online 12 February 2016

KEYWORDS

Couple stresses;
Variable viscosity;
Entropy generation;
Channel flow

Abstract This article addresses the influence of couple stresses in minimizing of entropy generation rate associated with heat transfer irreversibility in the steady flow of a variable viscous fluid through a channel with a non-uniform wall temperature. The flow is induced by a constant axial pressure gradient applied in the flow direction. It is assumed that the fluid viscosity varies linearly with temperature. Analytical expressions for the dimensionless equations governing the fluid velocity and temperature are derived and used to obtain expressions for volumetric entropy generation numbers, irreversibility distribution ratio and the Bejan number in the flow field. Plots for different pertinent parameters entering the velocity and temperature fields are displayed and discussed.
© 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

The major problem often encountered in energy generation is how to minimize or control energy wastages in form of heat dissipation. This has spurred a number of research works on the minimization of entropy generation especially when dealing with heat transfer problems. Of interest in this paper is the work done by Makinde [1] who gave a detailed thermodynamic analysis for a temperature dependent viscous fluid that is flowing steadily through a channel with non-uniform wall temperature. Interested readers can read the following papers for more interesting results on heat irreversibility in fluid flow based on second law to thermodynamics [2–20].

However, recent findings have shown that some fluids contain tiny polymer additives either to resist thermal effect as lubricating fluids in case of lubricants such as engine oil or to aid joint movement in the case of greasy synovial fluids. In human blood, tiny red blood cells are present for tissue respiration and many more real-life applications; therefore, a single constitutive equation cannot be used for non-Newtonian fluids. There a quite a lot of constitutive models are available in the literature to describe the rheological properties of these fluids but of interest in the present study is that introduced by Stokes' which takes into account the presence of couple stresses, body couples and non-symmetric stress tensor. This has been successfully used in the literature to describe fluid flow problems under different flow conditions [21–28] and references therein.

* Corresponding author.
E-mail address: adesanyas@run.edu.ng (S.O. Adesanya).
Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.01.011
1110-0168 © 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V.
This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

Motivated by studies in [29–33] the objective of the present study was to improve mechanics of the thermal system described in [1] so as to accommodate a wide range of non-Newtonian fluids used in many industrial and engineering set-ups exchanging heat between two thermal reservoirs. The rest of the paper is organized as follows. Section 2 presents the mathematical formulation and non-dimensionalization of the problem. In Section 3, the method of solution is described while Section 4 deals with the discussion of results based on the physics of the problem. Finally, Section 5 contains final remarks.

2. Mathematical formulation

Consider the steady flow of an incompressible, viscous couple stress fluid flow through infinite parallel plates with non-uniform wall temperature. The flow is induced by an axial pressure gradient. Then the equations governing the hydrodynamically and thermodynamically fully developed fluid flow are the momentum and energy equations [1,30–33]:

$$0 = -\frac{dP}{dx} + \frac{d}{dy'}\left(\mu'\frac{du'}{dy'}\right) - \eta\frac{d^4u'}{dy'^4} \quad (1)$$

$$0 = \frac{d^2T}{dy'^2} + \frac{\mu'}{k}\left(\frac{du'}{dy'}\right)^2 + \frac{\eta}{k}\left(\frac{d^2u'}{dy'^2}\right)^2 \quad (2)$$

the appropriate conditions at the walls are

$$\left.\begin{aligned} &T(0) = T_0,\ T(h) = T_1 \\ &u'(0) = \tfrac{d^2u'}{dy'^2}(0) = 0 = \tfrac{d^2u'}{dy'^2}(h) = u'(h) \end{aligned}\right\} \quad (3)$$

the linear variation of viscosity with temperature follows

$$\mu' = \mu_0(1 - \beta(T - T_0)) \quad (4)$$

Heat transfer to fluid flow with variable viscosity in a channel is irreversible. Hence, entropy production becomes continuous due to exchange of energy and momentum within the fluid particles in the channel. The expression for the total entropy generation within the fluid system can then be written as

$$E_G = \frac{k}{T_0^2}\left(\frac{dT}{dy'}\right)^2 + \frac{\mu'}{T_0}\left(\frac{du'}{dy'}\right)^2 + \frac{\eta}{T_0}\left(\frac{d^2u'}{dy'^2}\right)^2 \quad (5)$$

Introducing the following dimensionless variables and parameters

$$\begin{aligned} &y = \tfrac{y'}{h}, u = \tfrac{u'}{U}, \mu = \tfrac{\mu'}{\mu_0}, \theta = \tfrac{T - T_0}{T_1 - T_0}, \alpha = \beta(T_1 - T_0), \\ &\mathrm{Br} = \tfrac{\mu U^2}{k(T_1 - T_0)},\ a^2 = \tfrac{\eta}{\mu h^2},\ N_S = \tfrac{T_0^2 h^2 E_G}{k(T_1 - T_0)^2},\ \Omega = \tfrac{T_0}{T_1 - T_0} \end{aligned} \quad (6)$$

we get

$$\frac{d^4u}{dy^4} = a^2G + a^2\frac{d}{dy}\left((1 - \alpha\theta)\frac{du}{dy}\right) \quad (7)$$

$$\frac{d^2\theta}{dy^2} = -\mathrm{Br}\left\{(1 - \alpha\theta)\left(\frac{du}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2\right\} \quad (8)$$

$$N_S = \left(\frac{d\theta}{dy}\right)^2 + \frac{\mathrm{Br}}{\Omega}\left((1 - \alpha\theta)\left(\frac{du}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2\right) \quad (9)$$

the non-moving, stress-free upper wall of the channel is maintained at a given temperature

$$u(1) = \frac{d^2u}{dy^2}(1) = 0, \theta(1) = 1 \quad (10)$$

while the no-slip, stress-free conditions at the channel lower wall are also fixed and maintained at a temperature different from those of the upper wall,

$$u(0) = \frac{d^2u}{dy^2}(0) = 0 = \theta(0), \quad (11)$$

In Eqs. (1)–(11), y and y' – are dimensionless and dimensional distances measured in the normal direction respectively, h – is the channel width, and (u, u' U) – are the dimensionless, dimensional and characteristic velocity respectively. (μ, μ', μ_0) – represents the dimensionless dynamic fluid viscosity, dimensional dynamic fluid viscosity and referenced dynamic fluid viscosity respectively. (θ, T, T_1, T_0) – are the dimensionless fluid temperature, dimensional fluid temperature, upper wall fluid temperature and lower wall fluid temperature respectively, P is the pressure, α and β – are the dimensionless and dimensional viscosity-variation parameters respectively, Br – is the Brinkman number, k – is the thermal conductivity a^2 – dimensionless couple stress inverse parameter, η – couple stress parameter, (E_G, N_S) are the dimensional and dimensionless entropy generation rate and Ω is the temperature difference parameter.

3. Method of solution

As suggested in [1], we assume that $0 < \alpha << 1$ and as such it is convenient to obtain a perturbative solution in the form:

$$\left.\begin{aligned} \theta(y) &= \sum_0^1 \theta_n(y)\alpha^n + O(\alpha)^2, \\ u(y) &= \sum_0^1 u_n(y)\alpha^n + O(\alpha)^2. \end{aligned}\right\} \quad (10)$$

So that by substituting (10) in (7), (8) and equating coefficients, we get the following orders of

$$\begin{aligned} &O(\alpha)^0 : \frac{d^4u_0}{dy^4} = a^2\left(G + \frac{d^2u_0}{dy^2}\right); \\ &\frac{d^2u_0}{dy^2}(0) = u_0(0) = 0 = \frac{d^2u_0}{dy^2}(1) = u_0(1), \\ &O(\alpha)^1 : \frac{d^4u_1}{dy^4} = a^2\left(\frac{d^2u_1}{dy^2} - \frac{du_0}{dy}\frac{d\theta_0}{dy} - \theta_0\frac{d^2u_0}{dy^2}\right); \\ &\frac{d^2u_1}{dy^2}(0) = u_1(1) = 0 = \frac{d^2u_1}{dy^2}(1) = u_1(1), \end{aligned} \quad (11)$$

together with

$$\begin{aligned} &O(\alpha)^0 : \frac{d^2\theta_0}{dy'^2} = -\mathrm{Br}\left\{\left(\frac{du_0}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u_0}{dy^2}\right)^2\right\}; \\ &\theta_0(0) = 0, \theta_0(1) = 1, \\ &O(\alpha)^1 : \frac{d^2\theta_1}{dy'^2} = \mathrm{Br}\{\theta_0\left(\frac{du_0}{dy}\right)^2 - 2\frac{du_0}{dy}\frac{du_1}{dy} - \frac{2}{a^2}\frac{d^2u_0}{dy^2}\frac{d^2u_1}{dy^2}\}; \\ &\theta_1(0) = 0, \theta_1(1) = 0. \end{aligned} \quad (12)$$

Download English Version:

https://daneshyari.com/en/article/816059

Download Persian Version:

https://daneshyari.com/article/816059

Daneshyari.com