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ORIGINAL ARTICLE

Non-Newtonian model study for blood flow through a tapered artery with a stenosis

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KEYWORDS

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Abstract The blood flow through a tapered artery with a stenosis is analyzed, assuming the blood as tangent hyperbolic fluid model. The resulting nonlinear implicit system of partial differential equations is solved analytically with the help of perturbation method. The expressions for shear stress, velocity, flow rate, wall shear stress and longitudinal impedance are obtained. The variations of power law index m, Weissenberg number We, shape of stenosis n and stenosis size δ are discussed different type of tapered arteries.

1. Introduction

Blood flow is now well known to the physiologists as one of the major mechanisms due to its applications in arterial mechanics. In particular, blood flows in arteries is an important field of research because arterial diseases are a major cause of death in most of western countries. In the recent past, several theoretical and experimental studies [1–5] have been carried out to analyze the arterial flow characteristics of blood. Chakravarty and Sannigrahi [6] developed a nonlinear mathematical analytically to study the flow characteristics of blood through an artery in the presence of multistenoses when it is subjected to whole body acceleration. The unsteady non-Newtonian blood flow and mass transfer in symmetric and non-symmetric stenotic arteries are numerically simulated by Valencia and Villanueva [7]. Effect of stenosis on solitary waves in arteries has been studied by Bakirtas and Demiray [8]. Myers and Capper [9] studied exponential taper in arteries, and an exact solution has been evaluated to see its effect on blood flow velocity waveforms and impedance. The pulsatile flow of blood through a catheterized artery is analyzed by Sankar [10], assumed the blood as a two-fluid model. As the seminal contribution to the study of shear thinning viscoelastic nature of blood, Thurston [11] developed an extended Maxwell model which is applicable to one-dimensional flow. Some researchers [12–18] investigated that for blood flowing through small vessels, there is erythrocyte-freeplasma (Newtonian) layer adjacent to the vessel wall and a core layer of a suspension of all erythrocytes (non-Newtonian). Further recent literature can be viewed through Refs. [20–25].

Motivated from the extensive literature available on blood flow through arteries, the purpose of the present investigation is to discuss the tangent hyperbolic fluid [19] model for blood flow through a tapered artery with mild stenosis. The governing equations along with the boundary conditions of stenosed symmetric artery have been solved by regular perturbation method. The expressions for velocity, resistance impedance, wall shear stress and shearing stress at the stenosis throat have

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been examined. The graphical behavior of different type of tapered arteries has been discussed at the end of the article.

2. Formulation of the problem

We are considering the cylindrical coordinates (r, θ, z) in which $(r = 0)$ as the axis of the symmetry of the tube. We are considering the flow of an incompressible hyperbolic tangent fluid of constant viscosity η_0 and density ρ in a tube having length L and take $\bar{u}$ and $\bar{w}$ are the velocity component in $\bar{r}$ and $\bar{z}$ direction respectively. The geometry of the stenosis which is assumed to be symmetric can be described as [4].

$$h(z) = d(z)\left[1 - \eta\left(b^{n-1}(z-a) - (z-a)^n\right)\right],$$
$$a \leqslant z \leqslant a + b,$$
$$= d(z), \quad \text{otherwise} \tag{1}$$

with

$$d(z) = d_0 + \xi z, \tag{2}$$

where $d(z)$ is the radius of the tapered arterial segment in the stenotic region, d_0 is the radius of the non-tapered artery in the non-stenotic region, ξ is the tapering parameter, b is the length of stenosis, $(n \geqslant 2)$ is a parameter determining the shape of the constriction profile and referred to as the shape parameter (the symmetric stenosis occurs for $n = 2$) and a indicates its location as shown in Fig. 1. The parameter η is given by

$$\eta = \frac{\delta^* n^{\frac{n}{n-1}}}{d_0 b^n (n-1)}, \tag{3}$$

where δ denotes the maximum height of the stenosis located at

$$z = a + \frac{b}{n^{\frac{n}{n-1}}}.$$

The equations governing the steady incompressible tangent hyperbolic fluid are given as

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{4}$$

$$\rho\left(\bar{u}\frac{\partial}{\partial \bar{r}} + \bar{w}\frac{\partial}{\partial \bar{z}}\right)\bar{u} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{\tau}_{\bar{r}\bar{r}}) + \frac{\partial}{\partial \bar{z}}(\bar{\tau}_{\bar{r}\bar{z}}) - \frac{\bar{\tau}_{\bar{\theta}\bar{\theta}}}{\bar{r}}, \tag{5}$$

$$\rho\left(\bar{u}\frac{\partial}{\partial \bar{r}} + \bar{w}\frac{\partial}{\partial \bar{z}}\right)\bar{w} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{\tau}_{\bar{r}\bar{z}}) + \frac{\partial}{\partial \bar{z}}(\bar{\tau}_{\bar{z}\bar{z}}). \tag{6}$$

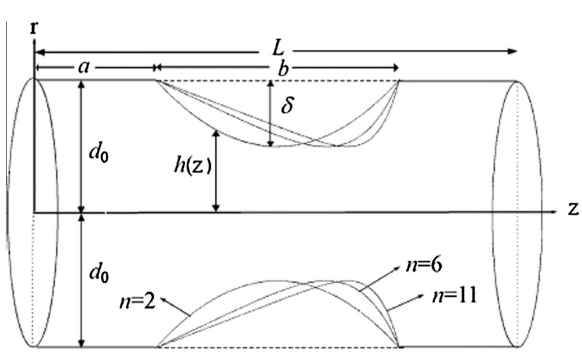

Figure 1 Geometry of the stenosis in the artery.

The constitutive equation for tangent hyperbolic fluid is defined as [19]

$$\widetilde{\mathbf{S}} = -P\mathbf{I} + \bar{\tau}, \tag{7a}$$

$$\bar{\tau} = \left[\left[\eta_\infty + (\eta_0 + \eta_\infty)\tanh\left(\Gamma\bar{\dot{\gamma}}\right)^m\right]\bar{\dot{\gamma}}\right], \tag{7b}$$

in which $\bar{\tau}$ is the extra stress tensor, η_∞ is the infinite shear rate viscosity, η_0 is the zero shear rate viscosity, Γ is the time constant, m is the power law index and $\bar{\dot{\gamma}}$ is defined as

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i\sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\Pi}, \tag{8}$$

where $\Pi = \frac{1}{2}trac\left(\text{grad } V + (\text{grad } V)^T\right)^2$. Here Π is the second invariant strain tensor. We consider the constitution Eq. (7), the case for which $\eta_\infty = 0$ because we cannot find the solution at the infinite shear rate viscosity. The component of extra stress tensor therefore, can be written as

$$\bar{\tau} = \eta_0\left[(\Gamma\bar{\dot{\gamma}})^m\right]\bar{\dot{\gamma}} = \eta_0\left[(1 + \Gamma\bar{\dot{\gamma}} - 1)^m\right]\bar{\dot{\gamma}}$$
$$= \eta_0\left[1 + m(\Gamma\bar{\dot{\gamma}} - 1)\right]\bar{\dot{\gamma}}. \tag{9}$$

Defining the non-dimensional variables

$$r = \frac{\bar{r}}{d_0}, \quad z = \frac{\bar{z}}{b}, \quad w = \frac{\bar{w}}{u_0}, \quad u = \frac{b\bar{u}}{u_0\delta}, \quad p = \frac{d_0^2\bar{p}}{u_0 b\eta_0}, \quad h = \frac{\bar{h}}{d_0}, \quad We = \frac{\Gamma u_0}{d_0},$$
$$Re = \frac{\rho b u_0}{\eta_0}, \quad \widetilde{\mathbf{S}}_{rr} = \frac{b\bar{\tau}_{rr}}{u_0\eta_0}, \quad \widetilde{\mathbf{S}}_{rz} = \frac{d_0\bar{\tau}_{rz}}{u_0\eta_0}, \quad \widetilde{\mathbf{S}}_{zz} = \frac{b\bar{\tau}_{zz}}{u_0\eta_0}, \quad \widetilde{\mathbf{S}}_{\theta\theta} = \frac{b\bar{\tau}_{\theta\theta}}{u_0\eta_0}, \tag{10}$$

where u_0 is the velocity averaged over the section of the tube of the width d_0.

Making use of Eqs. (9) and (10) into Eqs. (4)–(6), the appropriate equations describing the steady flow of an incompressible tangent hyperbolic fluid in the case of mild stenosis $\left(\frac{\delta^*}{d_0} \ll 1\right)$, subject to the additional conditions [4]

$$\text{(i)} \quad \frac{Re\,\delta^* n^{\left(\frac{1}{n-1}\right)}}{b} \ll 1, \tag{11}$$

$$\text{(ii)} \quad \frac{d_0 n^{\left(\frac{1}{n-1}\right)}}{b} \sim O(1), \tag{12}$$

can be written as

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{13}$$

$$\frac{\partial p}{\partial r} = 0, \tag{14}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left((m-1)\left(\frac{\partial w}{\partial r}\right) + Wem\left(\frac{\partial w}{\partial r}\right)^2\right)\right]. \tag{15}$$

The corresponding boundary conditions are

$$\frac{\partial w}{\partial r} = 0 \quad \text{at } r = 0, \tag{15a}$$
$$w = 0 \quad \text{at } r = h(z), \tag{15b}$$

where

$$h(z) = (1 + \xi z)[1 - \eta_1((z-\sigma) - (z-\sigma)^n)],$$
$$\sigma \leqslant z \leqslant \sigma + 1, \tag{16}$$