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ORIGINAL ARTICLE

Three dimensional peristaltic flow of hyperbolic tangent fluid in non-uniform channel having flexible walls

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Abstract In this present analysis, three dimensional peristaltic flow of hyperbolic tangent fluid in a non-uniform channel has been investigated. We have considered that the pressure is uniform over the whole cross section and the interial effects have been neglected. For this purpose we consider laminar flow under the assumptions of long wavelength ($\lambda \rightarrow \infty$) and creeping flow ($Re \rightarrow 0$) approximations. The attained highly nonlinear equations are solved with the help of Homotopy perturbation method. The influence of various physical parameters of interest is demonstrated graphically for wall tension, mass characterization, damping nature of the wall, wall rigidity, wall elastance, aspect ratio and the Weissenberg number. In this present investigation we found that the magnitude of the velocity is maximum in the center of the channel whereas it is minimum near the walls. Stream lines are also drawn to discuss the trapping mechanism for all the physical parameters. Comparison has also been presented between Newtonian and non-Newtonian fluid.

1. Introduction

In recent decades, peristaltic flow has gained a remarkable interest due to its lot of application in distinct field of sciences. In particular, it is mechanism of transporting the fluid in various biological systems. Several applications of peristalsis are transport of urine from kidney to bladder through ureter, swallowing food through esophagus, transport of spermatozoa induct efferent of male reproductive system tract, movement of ovum in female fallopian tube, swallowing of food through esophagus, transport of lymph in lymphatic vessels such as arterioles, capillaries, venules. An innumerable application of peristaltic pumping has been found in corrosive fluid or sensitive fluids, sanitary fluids, transport of slurries and noxious fluids in nuclear industry. After an impressive work of Lytham [1], many authors investigated analytically and experimentally the mechanism of peristalsis [2–20]. Shit and Roy [21] investigated peristaltic motion of couple stress fluid under the influence of hydromagnetic effects with the numerical scheme. His results depict that trapping fluid can be removed and the axial velocity can be decreased with the help of magnetic field. Mittra et al. [22] discussed the influence of wall properties and

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Nomenclature

u, w	velocity components in x and y directions in wave frame	ψ	stream function
x, y, z	Cartesian coordinate system	a	height of the channel
ρ	constant density	b	amplitude of the wave
Γ	time constant	Π	second invariant tensor
n	power law index	ϕ	amplitude ratio
Re	Reynolds number	d	width of the channel
β	aspect ratio	p	pressure
λ	wavelength	η_∞	infinite shear rate viscosity
c	velocity of propagation	η_0	zero shear rate viscosity
δ	ratio of height to wavelength	$\mathbf{S}$	stress tensor
		We	Weissenberg number

Poiseuille flow in peristalsis. He analyzed that the mean flow reversal existing at the boundaries and also at the center of the channel. He observed that due to elasticity in the walls of channel, there is flow reversal at the center of the channel.

Later, Mittra et al. [23] also studied the interaction of peristaltic motion with Poiseuille flow. He found that flow reversal is mainly dependent on the Poiseuille flow and the flow reversal is versatile from the center to the boundaries of the channel. Rashidi et al. [24] analyzed analytically the effects of heat transfer through a porous annulus with pulsating pressure gradient. Das et al. [25] numerically analyzed the variable fluid properties over permeable surface under the impact of thermophoretic Magnetohydrodynamics (MHD) slip flow. He assumed that magnetic field is a function of time and also he supposed that thermal conductivity and viscosity of the liquid vary as a linear and inverse function of temperature. He found that the thermal boundary layer thickness shows opposite behavior for viscosity parameter and surface convection parameter. Recently, Ellahi et al. [26] examined the mathematical analysis of peristaltic transport of an eyring powell fluid through a porous rectangular duct. Ellahi et al. [27] investigated the peristaltic flow in a non-uniform rectangular duct under the effects of heat and mass transfer. Recently Akram et al. [28] studied the Influence of lateral walls on peristaltic flow of a couple stress fluids in a non-uniform rectangular duct. Ellahi et al. [29] examined the peristaltic flow of Jeffrey in a rectangular duct under the effects of magnetic field through porous walls. He analyzed that due to the effects of magnetic field and porosity the velocity of the fluid decreases. Reddy [30] evaluated the influence of lateral walls on peristaltic flow in a rectangular duct. Riaz et al. [31] studied the peristaltic motion of three dimensional non-Newtonian Carreau fluid having compliant walls. He found that with the increment of parameter wall tension and mass characterization, the velocity of the fluid decreases whereas its behavior is opposite for the remaining parameters. He also observed that the fluid velocity is maximum at the middle of the channel. Shapiro et al. [32] studied the peristaltic pumping with long wavelengths at low Reynolds number. Rashidi et al. [33] described the pulsatile flow in a porous medium with the help of homotopy analysis method. Mekheimer [34] considered the peristaltic flow of blood under effect of a magnetic field in a non-uniform channels. Elnaby and Haroun [35] have investigated a new model for studying the effect of wall properties on peristaltic transport of a viscous fluid. Effects of hall currents on peristaltic transport with compliant walls were explored by Gad [36]. Mekheimer et al. [37] analyzed the endoscopic mechanism on peristaltic flow through a porous medium in an annulus. According to best of authors knowledge three dimensional peristaltic flow of hyperbolic tangent fluid in non-uniform duct of rectangular cross section with compliant walls has not yet been observed.

With above analysis in mind, we are interested in analytical approximation of peristaltic flow in non-uniform channel of rectangular cross section with compliant walls. We considered the flow under the assumptions of long wavelength and low Reynolds number approximation. The reduced highly nonlinear partial differential equations are solved with help of homotopy perturbation technique [38–42]. Homotopy perturbation method is an analytic method that is used to solve many peristaltic flow problems. Closed form solutions up to first order are presented. The impact of various pertinent parameters is plotted and discussed. The most interesting mechanism of peristalsis is trapping which is also taken into account by drawing stream of all the physical parameters.

2. Mathematical formulation

We consider the in-compressible hyperbolic tangent fluid in non-uniform duct of rectangular cross section. We have elected Cartesian coordinate system i.e. $x - axis$ is taken along the axial direction, $y - axis$ is taken along the lateral direction and $z - axis$ is taken along the vertical direction of the non-uniform channel as shown in Fig. 1.

The peristaltic waves on the walls can be written as [27]

$$\mathbf{H}(x,t) = \pm a + \mathbf{K}x \pm b\cos\frac{2\pi}{\lambda}(x - ct), \tag{1}$$

The walls parallel to xz plane are not interrupted and don't participate to any peristaltic wave motion. Let $(u, 0, w)$ be the velocity for the flow in a channel. The governing equations for the Hyperbolic Tangent fluid are defined as

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\mathrm{S}_{xx} + \frac{\partial}{\partial y}\mathrm{S}_{xy} + \frac{\partial}{\partial z}\mathrm{S}_{xz}, \tag{3}$$

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