Composites Part B 87 (2016) 12–20

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

Effective properties of regular elastic laminated shell composite

D. Guinovart-Sanjuán [a], R. Rodríguez-Ramos [a,*], R. Guinovart-Díaz [a], J. Bravo-Castillero [a], F.J. Sabina [b], J. Merodio [c], F. Lebon [d], S. Dumont [d,e], A. Conci [f]

[a] Facultad de Matemática y Computación, Universidad de La Habana, San Lázaro y L, Vedado, La Habana CP 10400, Cuba
[b] Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas, Universidad Nacional Autónoma de México, Apartado Postal 20-126, Delegación de Álvaro Obregón 01000, Mexico
[c] Departamento de Mecánica de los Medios Continuos y T. Estructuras, E.T.S. de Caminos, Canales y Puertos, Universidad Politécnica de Madrid, C.P. 28040 Madrid, Spain
[d] Laboratoire de Mécanique et d'Acoustique, Aix-Marseille University, CNRS, UPR 7051, Centrale Marseille, 4 Impasse Nikola Tesla, CS 40006, 13453 Marseille Cedex 13, France
[e] Université de Nîmes, Institut de Mathématiques Alexander Grothendieck, CNRS UMR 5149, CC. 051, Pl. E. Bataillon, 34 095 Montpellier Cedex 5, France
[f] Instituto de Computação/Universidade Federal Fluminense (UFF), Av. Gal. Milton Tavares de Souza, s/n°, Niterói, CEP: 24210-346 Rio de Janeiro, Brazil

ARTICLE INFO

Article history:
Received 27 December 2014
Received in revised form
25 September 2015
Accepted 27 September 2015
Available online 28 October 2015

Keywords:
A. Laminates
A. Layered structures
B. Mechanical properties
C. Analytical modelling
C. Numerical analysis

ABSTRACT

The manuscript offers a methodology to solve an analytical model of a heterogeneous elastic problem for curvilinear layered structures, using the two scales asymptotic homogenization method (AHM). The local problems and the mechanical properties of the local functions were derived. The analytical modeling for the linear elastic problem considering quasi-periodic multi-layered curvilinear composites and the corresponding homogenized problem were obtained. The analytic expression of the effective stress for curvilinear composites is presented. In order to validate the presented model, comparisons with a computational modeling and experimental results for Fibonacci laminated composite and wavy laminated structure are given. The methodology is applied to composites with thickness variation where the effective coefficients were computed and a comparison between the results reported by AHM and numerical analysis given by finite element method (FEM) is presented. Finally, the aorta is studied as a curvilinear laminated shell composite and the above results were used to determinate the effective elastic tensor for healthy and unhealthy aorta using AHM and FEM.

1. Introduction

The composite materials have been very popular for the last years due their exceptional mechanical properties. The laminated shell and sandwich composites had an important application in aerospace industry, automotive engineer [1–4] and textile manufacturing [5–7]. The composite conical shells have been widely used in various fields of technology as important structural components due to their special geometric shapes [8,9]. The use of the curvilinear composite structures in new engineering applications is significantly facilitated if the effective properties such as elastic, piezoelectic, thermo-elastic etc., can be predicted.

Homogenization is a useful mathematical method for solving boundary value problems in media with fine periodic structure. Two scale homogenization techniques have been used to solve periodic heterogeneous problems [10–12]. Other homogenization method is related to the concept of H-convergence [13,14]. The fourth order heterogeneous constitutive tensor H-converges to the effective tensor when the solution of the corresponding heterogeneous elasticity problem converges weakly to the effective displacement.

The development of new mathematical models and mathematic techniques helps to study the elastic properties of a laminated shell composite and bio-composite (cornea, aorta). Composites with periodic structure are often encountered in structural mechanics, in particular shell composites, [15–17]. The homogenization [10,18–21] and finite elements [7,22,23] techniques for shell are very used in these problems. Some recent works have been published related to effective properties of the smart composite shells [12,18,19] and wavy laminated shell composite [13,24], among others. In particular, behaviors of bio-composites are being analyzed with mathematical shell methods. Three dimensional models of human cornea [25,26], and the aorta [27], are important motivation for this work.

* Corresponding author. Tel.: +53 78790890.
E-mail addresses: reinaldo@matcom.uh.cu, rerora2006@gmail.com (R. Rodríguez-Ramos).

http://dx.doi.org/10.1016/j.compositesb.2015.09.051
1359-8368/

In this contribution, elastic composites in which the material coefficients are assumed to be rapidly oscillating periodic functions of a curvilinear coordinates system is studied and the two scales asymptotic homogenization method is used to solve the heterogeneous elastic problem. The novelty of this research is to obtain a homogenized problem with effective coefficients for a curvilinear laminated composites as extension of previous works [13,28–30]. Besides, the local problems and the general analytic expression for the effective coefficients are derived and the effective properties in Refs. [10,13] are obtained as particular case. The results presented in Ref. [31] for a Fibonacci laminated composite are compared with the results obtained by AHM for a quasi periodic shell composite. The effective coefficients reported in Ref. [13] for two waviness layers composite were compared with a laminated wavy structure with soft/hard interface between the layers. As an example of stratified structure, a composite with thickness variation is considered and the effective coefficients were computed using AHM (Asymptotic Homogenization Methods) and FEM (Finite Elements Method). The paper gives a methodology to analyze the heterogeneous elastic problem in curvilinear structures. This methodology is used to determinate the effective properties of the aorta and analyze the artery as a laminated shell composite. Comparisons between the effective coefficients for healthy and unhealthy (due to the presence of plaques) artery is presented. The present model is validated with Finite Element Method.

2. Elastic problem for curvilinear structure

2.1. Curvilinear structures

In the present work, a quasi-periodic elastic structure is understood, if there is a coordinate system $\mathbf{x} = (x_1, x_2, x_3) \in \Omega \subset \mathbb{R}^3$ such that the operator $\boldsymbol{\sigma} = F(\boldsymbol{\varepsilon}, \mathbf{x}, \mathbf{y})$ who related stress ($\boldsymbol{\sigma}$) and strain ($\boldsymbol{\varepsilon}$) is regular in $\mathbf{x}$ and $\mathbf{Y}$-periodic in $\mathbf{y}$, where $\mathbf{y} = \mathbf{x}/\varepsilon \in \mathbf{Y}$ ($\mathbf{Y}$ unit cell) and ε is a very small parameter.

Additionally, for certain structure there is an oscillation in its geometrical configuration and a stratified function is used to describe the geometry of the composite [13]. Also, in this sense a variation of the thickness of the unit cell can be considered. A generalization of these ideas is presented in further sections.

In the case when the operator $\boldsymbol{\sigma} = F(\boldsymbol{\varepsilon}, \mathbf{x}, \mathbf{y})$ is regular in $\mathbf{x}$ and $\mathbf{Y}$-periodic in $\mathbf{y}$, where $\mathbf{y} = \varrho(\mathbf{x})/\varepsilon$ and $\varrho : \mathbb{R}^3 \to \mathbb{R}^3$, it defines an elastic curvilinear structure.

The particular case when the function $\varrho \equiv \boldsymbol{I}$ (identity function) the curvilinear structure is a quasi-periodic structure. Another important example are the shell composites; for this one, the function ϱ, called stratified function, has the property $\varrho : \mathbb{R}^n \to \mathbb{R}^p$ where $n > p$, [13], see Fig. 1.

2.2. Statement of the elastic problem for curvilinear structures

Consider a certain curvilinear heterogeneous structure Ω bounded by $\Sigma = \Sigma_1 \cup \Sigma_2$ in the coordinate system $\mathbf{x} = (x_1, x_2, x_3)$ and a periodic function $\varrho = (\varrho_1(\mathbf{x}), \varrho_2(\mathbf{x}), \varrho_3(\mathbf{x}))$. The aforementioned operator F in this structure is the Hooke's law

$$\boldsymbol{\sigma}_\varepsilon = \mathbf{C}_\varepsilon\left(\frac{\varrho(\mathbf{x})}{\varepsilon}, \mathbf{x}\right) : \boldsymbol{\varepsilon}_\varepsilon, \tag{1}$$

where $\mathbf{C}_\varepsilon$ is regular in $\mathbf{x}$ and $\mathbf{Y}$-periodic in $\mathbf{y}$, where $\mathbf{y} = \varrho(\mathbf{x})/\varepsilon$ is the fast curvilinear coordinate system.

The equilibrium equation of the elastic theory has the following expression

$$\operatorname{div}\boldsymbol{\sigma}_\varepsilon + \mathbf{f} = 0 \quad \text{on} \quad \Omega, \tag{2}$$

with boundary conditions

$$\mathbf{u}^\varepsilon = \mathbf{u}^0 \text{on } \Sigma_1, \quad \boldsymbol{\sigma}^\varepsilon \cdot \mathbf{n} = \mathbf{S} \text{ on } \Sigma_2, \tag{3}$$

where $\mathbf{u}^\varepsilon$ is the displacement, $\mathbf{u}^0$ is the known displacement on Σ_1, $\mathbf{S}$ is the stress vector on Σ_2 and $\mathbf{n}$ is the external normal vector of Σ_2.

The coordinate system $\mathbf{x}$ is curvilinear and considering the Einstein's summation rule, where the Latin index run from 1 to 3, the expression of the equilibrium equation can be written

$$\sigma^{ij}\Big|\Big|_j + f_i = \sigma^{ij}_{,j} + \Gamma^i_{jk}\sigma^{kj} + \Gamma^j_{jk}\sigma^{ik} + f^i = 0 \quad \text{on} \quad \Omega, \tag{4}$$

where $\{\cdot\}\Big|\Big|_j$ denotes the covariant derivate, $\{\cdot\}_{,j} = \frac{\partial}{\partial x_j}\{\cdot\}$ the derivation respect to the slow or global curvilinear coordinate and Γ^i_{jk} the Christoffel's symbols of second type.

Considering the Cauchy's formula, who related the strains and the displacements $\varepsilon_{mn} = 1/2(u_m||_n + u_n||_m)$ and substituting into (1) yields

$$\sigma^{ij} = C^{ijmn}u_m||_n = C^{ijmn}\left(u_{m,n} - \Gamma^k_{mn}u_k\right). \tag{5}$$

Replacing (5) into (4), the generalized equilibrium problem of elasticity theory takes the following expression

$$\left(\frac{\varrho_{k,j}}{\varepsilon}C^{ijmn}_{|k} + C^{ijmn}_{,j} + \Gamma^i_{jk}C^{kjmn} + \Gamma^j_{jk}C^{ikmn}\right)\left(u_{m,n} - \Gamma^r_{mn}u_r\right) + C^{ijmn}\left(u_{m,nj} - \Gamma^r_{mn,j}u_r - \Gamma^r_{mn}u_{r,j}\right) + f^i = 0 \quad \text{on} \quad \Omega, \tag{6}$$

with boundary conditions

$$u_i = u^0_i \text{ on } \Sigma_1 \quad \left(C^{ijmn}\left(u_{m,n} - \Gamma^r_{mn}u_r\right)\right)n_j = S^i \text{ on } \Sigma_2, \tag{7}$$

where $\{\cdot\}_{|j} = \frac{\partial}{\partial y_j}\{\cdot\}$ denotes the derivation respect to the fast or local curvilinear coordinate.

3. Asymptotic homogenization method

In order to solve the problem (6) and (7) with fast oscillating coefficients, the two scales Asymptotic Homogenization Method (AHM) is used.

The general asymptotic expansion is

$$u^{(\varepsilon)}_m = v_m + \varepsilon \cdot v_l||_k N^{lk}_m + \varepsilon^2 \cdot \neq v_l||_k j N^{lkj}_m + \ldots, \tag{8}$$

where $v_m \equiv v_m(\mathbf{x})$, and the local functions $N^{lk\cdots}_m \equiv N^{lk\cdots}_m(\mathbf{x}, \mathbf{y})$ are $\mathbf{y}$-periodic functions and $\langle N^{lk\cdots}_m \rangle = 0$, where $\langle \varphi \rangle = \frac{1}{\mathbf{V_Y}} \int_{\mathbf{Y}} \varphi \sqrt{g} dy$; $\mathbf{V_Y}$ volume of $\mathbf{Y}$, $\sqrt{g} = \sqrt{\mathbf{det}[g_{ij}]}$ and $[g_{ij}]$ is the metric tensor [16].

In order to obtain the effective coefficients, the following expansion is proposed,