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Analytical investigation on free vibration of circular double-layer graphene sheets including geometrical defect and surface effects

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Effects of a single vacancy defect or a pin hole on free vibration behavior of a double layer graphene sheet are investigated. Using the nonlocal continuum theory as well as the Gurtin–Murdoch theory, the nonlocality and surface effects are considered in equations of motion. Both of in-phase and anti-phase vibration modes are analytically analyzed. Employing the translational addition theorem for cylindrical vector wave functions, the geometrical defect as a circular hole in arbitrary size and location is modeled. The van der Waals interaction between the upper and lower layers is included using the Lennard–Jones pair potential. The computational efficiency and accuracy of results are validated by literature. Effects of boundary conditions, geometrical properties, nonlocality and surface effect parameters on in-phase and anti-phase vibrational modes are investigated. Results reveal that the fundamental natural frequency of an annular double-layer graphene sheet with a free eccentric circular defect is less affected by the size and location of the defect. Moreover, the surface effect parameters have more significant effects on the in-phase vibration modes than the anti-phase ones.

1. Introduction

Graphene is one of the most important nano-sized structural elements which has been commonly used as component in nanobiological and nanoelectromechanical systems such as actuators, optoelectronics and biosensors during the last decades [1–3]. Its vibrational properties play an important role in structural stability of nano based systems used in dynamic environments.

Graphite consists of stacked layers of graphene sheet (GS) separated by 0.3 nm and held together by weak van der Waals (vdW) forces. It has extremely high strength and stiffness, and remarkable electronic and thermal conductivity along the basal plane. Due to production process of multilayer GSs, these may be opposed to structural defects. Some of the defects can be modeled as an eccentric hole. Hence, static and dynamic analyses of multilayer GSs with an eccentric defect are very important.

Due to shortcomings of classical theories in description of the proper mechanical behavior of nanostructures, the researchers have attempted to provide appropriate models and theories for this purpose. Departure of results of the well-known classical theories from those of experimental studies at nanoscales must return to effects of additional material properties or constitutive equations of nanomaterials [4]. It is shown that there are some additional material properties on external boundary layers of elastic media that are different from those of bulk materials. This concept is demonstrated by satisfaction of equilibrium equations at both surface and bulk materials by Gurtin and Murdoch through a continuum approach [5]. The surface properties cannot be overlooked in the study of nanostructures and nanomaterials due to the large value of surface to volume ratios at this scale. Basically, since the surface to volume ratio of an elastic boy decreases at larger scales, the surface effects decrease and the concept of size dependent mechanical behavior of nanostructures is introduced [6]. The surface of a solid is a region with small thickness which has different properties from the bulk. If the surface energy-to-bulk energy ratio is large, for example in the case of nanostructures, the surface effects cannot be ignored [7]. To account the effect of surfaces/interfaces on mechanical deformation, the surface elasticity theory is presented by modeling the surface as a two dimensional membrane adhering to the underlying bulk material without slipping [8].

In recent years, many various studies on the mechanical behavior of GSs, such as their buckling instability and vibrational characteristics have been carried out. Wang et al. [9] developed a nonlinear continuum model for the vibrational analysis of

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multilayer GSs, in which there are nonlinear vdW interactions between the two adjacent layers. Arash and Wang [10] investigated free vibration of single-layer graphene sheets (SLGSs) and double-layer graphene sheets (DLGSs) by employing nonlocal continuum theory and Molecular dynamic (MD) simulations. Pradhan and Phadikar [11] worked on the vibration analysis of DLGSs embedded in a polymer matrix based upon the nonlocal continuum mechanics. Wang et al. [12] studied the thermal effects on the vibration properties of the double-layered nanoplates. Kumar et al. [13] studied the thermal vibration analysis of DLGSs embedded in polymer elastic medium, using the plate theory and nonlocal continuum mechanics for small scale effects. Natsuki et al. [14] used circular plate theory to study the natural vibration of DLGSs. Mohammadi et al. [15] studied free transverse vibration analysis of circular and annular graphene sheets with various boundary conditions using the nonlocal continuum plate model. They derived governing equations by using the nonlocal elasticity theory for SLGS. Analytical frequency equations for circular and annular graphene sheets were obtained based on different cases of boundary conditions. Hashemi et al. [16] developed an exact solution for free vibration of coupled double viscoelastic graphene sheets by viscopasternak medium. Mohammadi et al. [17] investigated the shear buckling of orthotropic rectangular graphene sheet embedded in an elastic medium in thermal environment. Nonlocal elasticity theory had been implemented to investigate the shear buckling of orthotropic SLGSs in thermal environment. Ansari et al. [18] presented nonlinear analysis of forced vibration of nonlocal third-order shear deformable beam model of magneto-electro-thermo elastic nanobeams. The magneto-electro-thermo nanobeam was assumed to be subjected to the external electric potential, magnetic potential and constant temperature rise. Jomehzadeh and Pugno [19] presented bending stiffening of graphene and other 2D materials via controlled rippling. The initial rippling of the surface was modeled by cosine functions with a hierarchical topology. Considering both large displacement and small scale effect, the governing equilibrium equations were determined and solved.

For vibration characteristics of DLGSs, the upper and the lower layers of DLGSs deflect in the same or opposite direction, and are defined as the in-phase mode (IPM) and anti-phase mode (APM), respectively. Because this is a feature of DLGSs, it is important to study vibration characteristics of DLGSs through the consideration of the IPM and APM.

In the present work, for the first time, an analytical approach is employed to analyze IPM and APM of a defective circular DLGS. The nonlocal continuum theory as well as the Gurtin–Murdoch theory of elastic solid surfaces is used to obtain equations of motion of the DLGS. The DLGS has a circular perforation as geometrical defect with arbitrary size and location. The upper and lower layers of the circular DLGSs are held together by vdW forces which can be obtained from the Lennard-Jones pair potential [22]. The translational addition theorem is used to model the eccentric circular hole. To validate the accuracy of the present approach, the results are compared with the solutions found in the literature. Moreover, the effects of small scale coefficient, surface effect parameters, eccentricity and radius of internal hole on the in-phase and anti-phase frequencies of the DLGSs are examined.

2. Geometrical configuration

A circular DLGS with an eccentric hole and upper and lower surface layers is depicted in Fig. 1. The graphene sheet's geometric properties are denoted by inside radius R_2, outside radius R_1, eccentricity ε, and thickness h. The elastic modulus, Poisson's ratio and mass density of the bulk part of DLGS are respectively indicated by E, υ and ρ. The parameters λ and μ indicate the classical Lame

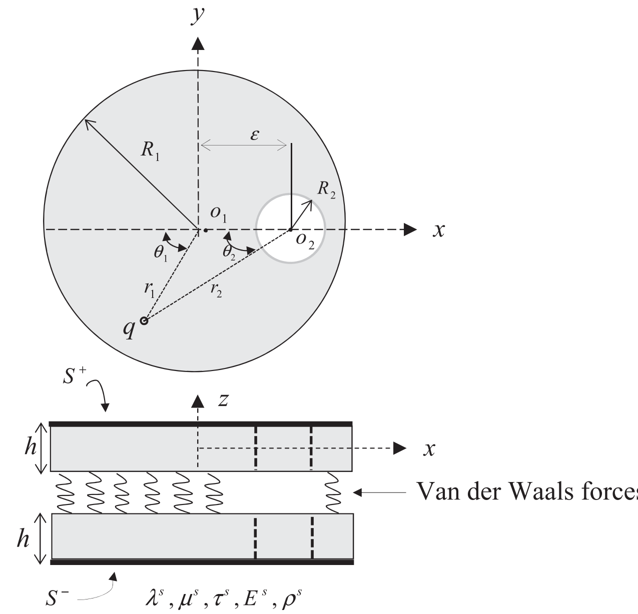

Fig. 1. Schematic of a circular double-layer graphene sheet with upper and lower thin skin layers carrying surface effects.

constants while the surface Lame constants are shown by λ^s and μ^s. τ^s is the surface residual stress which is uniformly distributed on the upper and lower surfaces of the DLGS. The surface mass density and the surface elastic modulus of the DLGS are also indicated by ρ^s and E^s, respectively. To interpret mathematical formulations, two polar coordinates (r_1, θ_1) and (r_2, θ_2) are taken to coincide with the center of the outer and inner circles, respectively. The upper and lower surfaces of the DLGS at $z = \pm\frac{h}{2}$ are denoted by s^+ and s^-, respectively.

3. The equations of motion

Based on nonlocal theory presented by Eringen [20], the constitutive equation will be obtained as

$$\left(1 - \mu\nabla^2\right)\sigma^b = C : \varepsilon \tag{1}$$

here ∇^2 is Laplacian operator and $\mu = (e_0 a)^2$ where a is internal characteristic length which depends on lattice parameter, granular size and C–C bonds and e_0 is a constant correlated by material type. The parameter e_0a is known as small scale. The superscript b indicates bulk part of the DLGS. C is the elastic modulus tensor. The displacement components (u,v,w) for an isotropic circular plate according to the classical plate theory can be written in a general form as

$$u(r_2,\theta_2,z) = -z\frac{\partial w(r_2,\theta_2)}{\partial r_2} \tag{2.1}$$

$$v(r_2,\theta_2,z) = -z\frac{1}{r_2}\frac{\partial w(r_2,\theta_2)}{\partial \theta_2} \tag{2.2}$$

$$w(r_2,\theta_2,z) = w(r_2,\theta_2) \tag{2.3}$$

Using Eq. (2), the strain components in the polar coordinate (r_2, θ_2) can be derived as