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Theoretical and numerical predictions of stretch-bending deformation behavior in composite sheet

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ABSTRACT

Stretch-bending of fiber reinforced composite sheets is one of the thermo-mechanical forming techniques in composite materials manufacturing. Due to an anisotropic nature of composite materials, predicting their deformation behavior during combined stretch-bending is difficult. To analyze this behavior, a mathematical model for semi-cylinder forming of continuous and unidirectional fiber reinforced composite sheet is developed. Plies in the sheet are treated as continuum bodies that interact through weak viscous interfaces. The model depicts both spatial and temporal variations of stresses and strains in composite sheet. Based on this model, numerical examples on semi-cylinder forming of cross-ply and angle-ply symmetric composite sheets are given and the results show good agreement with the simulation results obtained by ABAQUS commercial software.

1. Introduction

Fiber reinforced composite materials provide many advantages especially in automobile and aerospace industries where light weight, high stiffness, and toughness are the primary concerns. The applicability of these composite materials ranges from secondary to primary components in both automobiles and aircrafts. Boeing 787 (Dreamliner) and Airbus 380 are some of the noteworthy aircrafts that use, dominantly, carbon-fiber and glass-fiber reinforced composite materials. Though the demand for reinforced composite materials is increasing, there are critical limitations to reach mass production by the conventional manufacturing processes such as hand-layup, pultrusion, compression and curing in autoclave facilities. Stamping is found to be an alternative technique that reduces time consumption for desired composite parts production.

However, the prized advantages of composite materials such as light weight, high stiffness and toughness are limited by their formability into different shapes. Since stamping is performed at elevated temperature, misalignment of reinforcing fibers may occur [1,2]. Furthermore, the unbalanced extensibility of stacked plies renders inter-ply shear deformation that causes buckling and wrinkling within the deformed part in response to in-plane or out-of-plane loading [3]. In order to reduce this inter-ply shear deformation, the plies have to slip over each other [4,5]. Forming defects such as misalignment, buckling and wrinkling of reinforcing fibers can adversely affect the mechanical properties of composite materials. Hence, research activities in field of reinforced composite forming focus on how to avoid such defect and, thereby, retain the privileged advantages of composite materials.

During stamping, each ply in the sheet experiences unique deformation behavior based on its respective fiber orientation. This results in complex deformation phenomena throughout forming processes. Understanding, the spatial and temporal variation of strain and stress fields during composite forming is essential to predict the progress of the deformation behavior.

In this study, the stretch-bending process for shaping flat composite sheet into semi-cylindrical configuration was considered. To author's knowledge, there is no previous work on theoretical formulation for semi-cylinder forming of unidirectional and continuous fiber reinforced composite sheet. Since composite forming is conducted at elevated temperature inter-ply viscosity reduces invoking inter-ply slippage. This weak inter-ply interaction phenomena is also incorporated in the mathematical formulation. In fact, Tam and Gutowski had developed the mathematical formulation for ply-slip deformation during V-bending of composite part [4]. Our current work is somehow acquainted with

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Nomenclature

$[\Lambda]$	singular value
$[S]$	left unitary matrix
$[S^{-1}]$	right unitary matrix
$\overline{Q}^{\,i}_{\alpha\beta}$	$\alpha\beta$ component of reduced stiffness of i^{th} ply
δ_i	thickness of inter-ply resin-rich layer at i^{th} interface
$\dot{U}^i$	velocity of i^{th} ply along circumferential direction
U^i	displacement of i^{th} ply along circumferential direction
μ_i	inter-ply shear viscosity at i^{th} interface
σ^i_θ	stress on i^{th} ply along circumferential direction
$\tau^i_{\theta r}$	inter-ply shear stress at i^{th} interface
ε^i_θ	strain of i^{th} ply along circumferential direction
C^i	right Cauchy-Green deformation tensor of i^{th} ply
F^i	deformation gradient tensor of i^{th} ply
α^i	inter-ply shearing factor of i^{th} ply
γ^i	in-plane shearing factor of i^{th} ply
λ^i_θ	stretch ratio of i^{th} ply in circumferential direction
λ^i_z	stretch ratio of i^{th} ply in axial direction
R_C	radius of curvature of lower die fillet
R_P	radius of forming punch

their analytical formulation procedure. During stamping, the blank is subjected to rigid rotation, simple shear and stretch motion. Hence, in subsequent sections, the word "stretch-bending" is used as an alternative to "stamp forming". The sheet was clamped in its two opposite width-wise edges while the remaining two opposite edges are free. To analyze deformation behavior in composite sheet, a mathematical model is derived in section two. Section three explains the performed computer simulation on cylindrical forming to verify the fidelity of the developed mathematical model.

2. Mathematical model

Semi-cylinder shape forming of composite materials is one of the interest in aircraft manufacturing industries. For instance, Sikken-type stiffener, made of reinforced composite part, is one of the components in use in the wing and fuselage assemblies of aircrafts. This component has geometry of semi-cylindrical curve whose both ends are quarters of a sphere as shown in Fig. 1. The formability of composite materials into Sikken-type structure is limited by occurrence of wrinkles around bending regions [6,7]. During stamping, wrinkles are observed near, both, semi-cylindrical and spherical curves [8]. In this work, mathematical model for the semi-cylindrical segment of Sikken-type structure is developed to analyze deformation and stress evolution during the stretch-bending operation.

The overall configuration of the composite sheet after a punch is displaced downward by Δ_i increment is shown in Fig. 2(a). The sheet is segmented into three regions of distinct deformation behaviors. Two successive deformation configurations of the region ***II*** is shown in Fig. 2(b). In the subsequent mathematical development these deformation characteristics will be taken into consideration. Also, the plies are treated as deformable bodies that interact through relatively weak viscous interface. Fig. 3(a) shows three arbitrarily chosen successive layers from a segment of the composite sheet that conforms to the punch geometry. Fig. 3(b) describes the stress components and the geometric parameters on a differential element taken out of the middle (i^{th}) ply of the selected successive layers. In this region, once the sheet is conformed to the geometry of the punch, its motion has a combined effect of: rigid rotation about fixed coordinate system, simple shear, and stretch in circumferential direction. For convenience, a cylindrical coordinate system is taken in which the circumferential, axial, and out-of-plane transverse directions of the sheet are along $\boldsymbol{\theta}$, $\boldsymbol{z}$ and $\boldsymbol{r}$ axes, respectively.

The transverse shear stresses on the upper ($\tau^{i-1}_{\theta r}$) and lower ($\tau^i_{\theta r}$) surfaces of the differential element are expressed, in terms of relative circumferential velocities of adjacent plies, as follows;

$$\tau^{i-1}_{\theta r}=\frac{\mu^{i-1}}{\delta^{i-1}}\left[\dot{U}^{i-1}-\dot{U}^{i}\right] \tag{2.1}$$

and

$$\tau^{i}_{\theta r}=\frac{\mu^{i}}{\delta^{i}}\left[\dot{U}^{i}-\dot{U}^{i+1}\right] \tag{2.2}$$

where U^{i-1}, U^i, U^{i+1} are circumferential displacements of the $(i-1)^{\text{th}}$, i^{th}, $(i+1)^{\text{th}}$ plies, respectively.

For the differential element under static equilibrium, Eq. (2.3) can be obtained as

$$\sum F_\theta=(tw)\left[\sigma^i_\theta+\frac{\partial\sigma^i_\theta}{\partial\theta}d\theta\right]-(tw)\sigma^i_\theta-wr^i d\theta\tau^i_{\theta r}+wr^{i-1}d\theta\tau^{i-1}_{\theta r}=0 \tag{2.3}$$

where $r^i=R_p+it$ and $r^{i-1}=R_p+(i-1)t$ are the radial distances from the punch center to the i^{th} and $(i-1)^{\text{th}}$ plies, respectively.

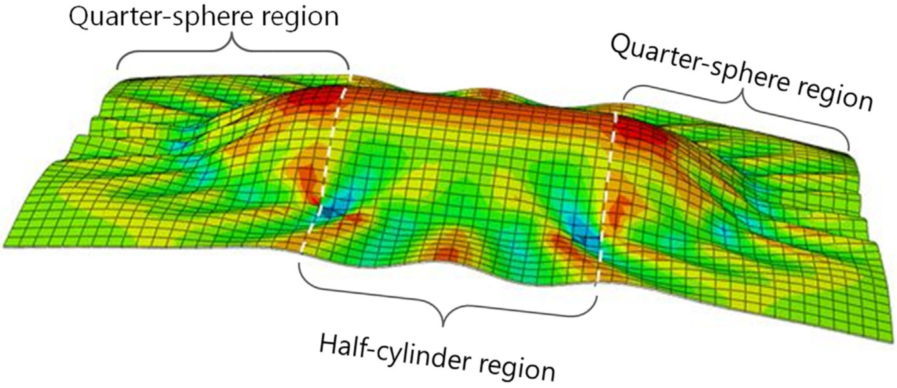

Fig. 1. Geometry of Sikken-type stiffener.