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Simplified ultimate strength analysis of compressed composite plates with linear material degradation

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ABSTRACT

Simply supported, rectangular, composite plates subjected to in-plane compressive load have been investigated for ultimate strength. An efficient, semi-analytical method has been established based on large deflection theory and first order shear deformation theory. After damage initiation, linear degradation of the material properties has been applied to the affected region of a failed ply. Two different displacement fields have been examined for their influence on the strength predictions. The approach is validated against earlier advanced finite element calculations, and can be readily applied in specific design situations or to generate parametric design curves.

1. Introduction

Composite plates are used as structural components in many large structures, such as naval ships and wind turbine blades. In design of composite structures, buckling analysis is very often confined to estimation of elastic critical loads without taking account of geometric imperfections or material degradation. For many plates, the elastic critical loads give very conservative solutions, which in turn prevent the full utilisation of the material, while for others the neglect of imperfections can lead to unsafe estimates of strength. In contrast to composite plates, for plates made of steel, design strength curves that take account of slenderness and geometric imperfections have been established based on extensive studies. At present, ultimate strength analysis of composite structures can be performed using nonlinear finite element (FE) methods. However, such analyses are time consuming to perform and impractical for most design purposes. For strength predictions of stiffened, thin steel plates under in-plane loading, Brubak et al. [1–3] have developed several simplified semi-analytical methods.

The present paper reports on the latest developments in a study that aims to extend these efficient methods to fibre-reinforced composite plates, taking account of (i) appropriate failure and degradation models for composites, (ii) initial geometric imperfections, (iii) out-of-plane shear deformations and (iv) post-buckling deformations. A series of simplified models based on small deflection theory, presented in [4], showed that neglect of post-buckling behaviour makes the ultimate strength predictions very conservative, especially for thin plates. Several models based on large deflection theory, developed in [5] and reported in more detail in [6], give significant improvements compared to those in [4]. However, the models proposed in [5] were implemented with instantaneous material degradation once a given failure criterion was reached, and this resulted in significant underestimation of the strength for some of the cases. The models in the present paper adopt a linear degradation of the material properties. The degradation approach, in which the material stiffness reduction is limited to the affected regions of a failed ply, is developed in combination with the Hashin and Rotem failure criterion [7]. To validate the method, the results are compared with the FE analyses conducted by Misirlis using ABAQUS, reported by Hayman et al. [8].

2. Boundary conditions and displacement fields

A rectangular plate is considered, with dimensions $a \times b$ (Fig. 1) and an initial out-of-plane deformation w_{init}. The plate is simply supported on all edges and subjected to a mean compression N_x in the x-direction. In the analyses, this is achieved by restraining the edge $x = 0$ in the x-direction and applying a uniform, negative displacement u_c in the x-direction on the edge $x = a$, all four edges being held straight. The total out-of-plane deformation is

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$w_{tot} = w_{init} + w$. Two alternative displacement fields, designated DF1 and DF2, are assumed. In DF1, each deformation component is assumed in the form of a truncated double Fourier series [3,9], the in-plane displacements having in addition a linear component [3,10]:

$$u_0(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} u_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) + u_c \frac{x}{a} \tag{1a}$$

$$v_0(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} v_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) + v_c \frac{y}{b} \tag{1b}$$

$$\phi_x(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} x_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \tag{1c}$$

$$\phi_y(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} y_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \tag{1d}$$

$$\begin{aligned} w_{tot}(x,y) &= w(x,y) + w_{init}(x,y) \\ &= \sum_{n=1}^{N}\sum_{m=1}^{M} w_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \\ &\quad + \sum_{n=1}^{N}\sum_{m=1}^{M} w_{imn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \end{aligned} \tag{1e}$$

The symbols u_0 and v_0 represent the mid-plane displacements in the x- and y-directions, respectively. The rotations of a transverse normal about axes parallel to the y and x axes are denoted by ϕ_x and ϕ_y, respectively. The coefficients u_c, v_c, u_{mn}, v_{mn}, x_{mn}, y_{mn} and w_{mn} are unknowns, w_{imn} are given imperfection amplitudes, and m, n, M and N are positive integers.

In the alternative displacement field DF2, the in-plane displacement fields, Eqs. (1a) and (1b), are replaced by those used by Reddy [11] for anti-symmetric angle-ply laminates:

$$u_0(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} u_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) + u_c \frac{x}{a} \tag{2a}$$

$$v_0(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} v_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) + v_c \frac{y}{b} \tag{2b}$$

Note that all four edges are still constrained to remain straight, but u_0 now varies differently along the edges $y = 0,b$ and v_0 varies differently along $x = 0,a$.

3. Methodology

3.1. Introduction

The semi-analytical method is based on large deflection theory combined with the first order shear deformation theory. The load–displacement response is traced by an incremental procedure, where an arc length parameter is used as a propagation parameter [12]. This method is presented in detail in Yang [6]. A brief review of the background theory and the methodology are provided in Sections 3.2 and 3.3.

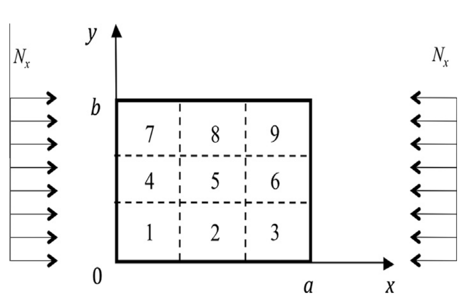

Fig. 1. Plate geometry and load condition. The broken lines and the numbers are explained in Section 5.

3.2. Kinematics

For a plate with an out-of-plane imperfection w_{init} and additional out-of-plane displacement w, the nonlinear strains become [11,13]:

$$\varepsilon_{xx} = \varepsilon_{xx}^0 + z\kappa_{xx} = \left(\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w_{init}}{\partial x}\right) + z\left(\frac{\partial \phi_x}{\partial x}\right) \tag{3a}$$

$$\varepsilon_{yy} = \varepsilon_{yy}^0 + z\kappa_{yy} = \left(\frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\partial w}{\partial y}\frac{\partial w_{init}}{\partial y}\right) + z\left(\frac{\partial \phi_y}{\partial y}\right) \tag{3b}$$

$$\begin{aligned} \gamma_{xy} &= \gamma_{xy}^0 + z\kappa_{xy} \\ &= \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w_{init}}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial w_{init}}{\partial x}\right) \\ &\quad + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) \end{aligned} \tag{3c}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \phi_x \tag{3d}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \phi_y \tag{3e}$$

Here x and y are the in-plane coordinates and z is the distance from the middle plane of the plate. The terms with the super index "0" denote the mid-plane membrane strains, while κ are the curvatures.

3.3. Arc length method

Using the Rayleigh–Ritz method and denoting the total potential energy as Π, the incremental form of the stationary potential energy condition for equilibrium is $\delta\dot{\Pi} = 0$, and thus

$$\frac{\partial \dot{\Pi}}{\partial \lambda_i} = \frac{\partial}{\partial \eta}\left(\frac{\partial \Pi}{\partial \lambda_i}\right) = C_{ij}\dot{\lambda}_j + F_i\dot{\Lambda} = 0 \tag{4}$$

Here, λ_i represents the displacement and rotation amplitudes, while Λ is the load parameter. A dot above a symbol means differentiation with respect to an arc length parameter η. Further, C_{ij} is a generalised, incremental stiffness matrix and $F_i\dot{\Lambda}$ is a generalised, incremental load vector, where i indicates the row number and j the column number in a matrix. The additional equation required to solve the problem is given by relating the arc length increment parameter $\Delta\eta$ to the load increment $\Delta\Lambda$ and the incremental displacement amplitude $\Delta\lambda_i$:

$$\dot{\Lambda}^2 + \sum_{Displ.} \frac{\dot{\lambda}_i^2}{t^2} + \sum_{Rot.} \dot{\lambda}_i^2 = 1 \tag{5}$$

where t is the plate thickness.

In the propagation process, $\dot{\Lambda}^s$ and $\dot{\lambda}_j^s$ can be determined from Eqs. (4) and (5) at stage s. The solutions at the next stage $(s+1)$ are then obtained by the first order Taylor series expansion:

$$\lambda_j^{s+1} = \lambda_j^s + \dot{\lambda}_j^s \Delta\eta \tag{6a}$$

$$\Lambda^{s+1} = \Lambda^s + \dot{\Lambda}^s \Delta\eta \tag{6b}$$