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Effects of nonlinear strain components on the buckling response of stiffened shear-deformable composite plates

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ABSTRACT

A detailed investigation of the weight of each non linear term of the Green–Lagrange strain displacement equation is presented, with reference to the buckling of orthotropic, both flat and prismatic, Mindlin plates. Usually in the literature, in buckling analysis only the second order terms related to the out-of-plane displacement are considered. Such heuristic simplification, known as von Kármán hypothesis, starts by the consideration that the buckling mode of a flat plate is described by dominant out-of-plane displacement and disregards the non-linear terms of the Green–Lagrange strain tensor depending on the in plane displacement components, whose role is confined to first order, say pre-critical, deformation. The present paper shows that disregarding the non linear terms related to the in-plane strain–displacement is equivalent to neglect shear induced rotation. In the work, the governing equations are derived using the principle of strain energy minimum and the differential equations solution is gained by using the general Levy-type method. The obtained results show that the von Kármán model overestimates the critical load when, in buckling mode, magnitudes of shear rotation, in-plane and out-of-plane displacements are comparable.

1. Introduction

Structures made of laminated composite materials are increasingly used in many engineering branches, especially when the weight of the structure is a significant factor to be considered in the analysis and design of structural members. Nevertheless, such structures can be affected to a buckling failure, and the growing number of papers published on the subject show how the definition of a model able to predict their critical behavior is again considered an open question. Indeed, starting from the nonlinear theory of the elasticity, the buckling analysis of elastic structures requires some kinematical assumptions able to get simplified structural models, concerning two-dimensional as well as one-dimensional bodies [1].

The simplest two-dimensional model can be obtained according to the Kirchhoff–Love hypotheses [2] that, neglecting the effect of the transverse shear deformation, provides reasonable results for thin plates [3,4], and shells [5]. However, the Kirchhoff's theory generally overestimates the buckling loads of thick plates, for which the transverse shear becomes effective. The first-order shear deformation plate theory, proposed by Reissner [6] and Mindlin [7], relaxes the assumption of normality of the cross-section and overcomes some of the intrinsic limitations of the Kirchhoff model [8]. However, due to the impossibility of fulfilling homogeneous boundary conditions on tractions on the limit planes of the plate, the Mindlin model is not consistent with respect to the mathematical theory of elasticity. Several corrections have been proposed in literature (see, for example [9,10] and, more recently [11–13]), involving higher-order terms of the Taylor expansion of the displacements in the thickness coordinate, i.e. assuming quadratic, cubic or higher variations of surface parallel displacements through the entire thickness of the laminates to address the "correct" structural behavior. An overview of the algebraic relationships between the buckling solutions of the classical plate theory and those of the first-order and third-order shear deformation plate theories can be found in [14].

Besides the choice of a proper structural model, an actual solution of the equations governing the buckling phenomenon require further approximations on the displacement field. The Finite Element Method (FEM) is the most powerful and most popular technique for computing accurate solutions of partial differential equations, since it can be adapted to problems of great complexity and unusual geometry, and it has been widely used for analyze the critical behavior of Kirchhoff [15] and Mindlin [16] plate.

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When structures are prismatic, constituted by flat or curved plate components rigidly connected along their longitudinal edges to form arbitrary cross-section profiles, the Finite Strip Method (FSM) represents a very competitive alternative to the FEM in terms of accuracy, computational and data preparation time [17]. The Boundary Element Method [18], the Superposition Method [19], and the Finite Difference Method [20] are numerical alternative for buckling analysis of both flat and stiffened plates.

Finally, the third aspect to be considered is related to the nonlinear model adopted for predicting the buckling load. In such matter the von Kármán plate theory is generally accepted within the scientific community. Starting by the heuristic consideration that the buckling mode of a flat plate is described by dominant out-of-plane displacements, the von Kármán model discards the non-linear terms of the Green–Lagrange strain tensor depending on in-plane displacement components, whose role is confined to first order, say pre-critical, deformation [21].

In previous works the author demonstrated that such non linear terms cannot be considered negligible in the stability analysis of prismatic structures consisting of a series of flat, rectangular isotropic [22] or orthotropic [23] thin plates. In particular, whenever the plate undergoes to global flexural or flexo-torsional buckling modes, the buckling deformed shape presents displacement that does not belong to any local out-of-plane direction, and the von Kármán model can greatly overestimate the related critical load.

In the present paper, a complete discussion on the influence of the nonlinear Green–Lagrange strain tensor terms on the buckling of orthotropic, moderately thick plates is presented. Starting by the Mindlin hypotheses, the equilibrium equations have been derived using the principle of strain energy stationarity. The obtained equations show how, for the Mindlin plate model, the nonlinear terms associated to the Green–Lagrange strain tensor influence directly the out-of-plane equations and, as consequence, the von Kármán hypothesis can be considered adequate only if pure out-of-plane buckling occurs, namely characterized by both negligible in-plane displacements and rotations.

In this paper, by means of the Levy-type method [24], a closed form solution, rigorously exact for plates with two opposite edges simply supported, has been derived for the ruling equations. In order to highlight the role of any of the non-linear terms usually neglected under the von Kármán hypothesis, such solution has been used to model the critical behavior of both flat and stiffened plates varying geometry, boundary and load conditions.

2. Buckling of Mindlin plates using Green–Lagrange strain tensor

Consider a rectangular orthotropic plate of dimensions (a, b), thickness h, referred to the rectangular coordinate system (O, x, y, z) indicate in Fig. 1, and subjected to uniform in-plane load in x and y direction. By using the Mindlin first order shear deformation plate theory the displacement components of the plate points, represented by the vector $\mathbf{s} = [s_x\ s_y\ s_z]^T$, can be represented in terms of the generalized local displacements (u, v, w) and of the shear rotations around the in-plane axes, say (φ_x, φ_y), of the mid-surface as follows:

$$\begin{aligned} s_x(x,y,z) &= u(x,y) - z\cdot\varphi_x(x,y) \\ s_y(x,y,z) &= v(x,y) - z\cdot\varphi_y(x,y) \\ s_z(x,y,z) &= w(x,y) \end{aligned} \tag{1}$$

The non-linear expressions for the strain components contributing to the strain energy can be put in the following form:

$$\begin{aligned} \varepsilon_x &= s_{x,x} + \frac{1}{2}\left(k_1 s_{x,x}^2 + k_2 s_{y,x}^2 + k_3 s_{z,x}^2\right) \\ \varepsilon_y &= s_{y,y} + \frac{1}{2}\left(k_1 s_{x,y}^2 + k_2 s_{y,y}^2 + k_3 s_{z,y}^2\right) \\ \gamma_{xy} &= s_{x,y} + s_{y,x} + k_1 s_{x,x}s_{x,y} + k_2 s_{y,x}s_{y,y} + k_3 s_{z,x}s_{z,y} \\ \gamma_{xz} &= s_{x,z} + s_{z,x} + k_1 s_{x,x}s_{x,z} + k_2 s_{y,x}s_{y,z} + k_3 s_{z,x}s_{z,z} \\ \gamma_{yz} &= s_{y,z} + s_{z,y} + k_1 s_{x,y}s_{x,z} + k_2 s_{y,y}s_{y,z} + k_3 s_{z,y}s_{z,z} \end{aligned} \tag{2}$$

In Eq. (2), and in the following, comma before subscripts indicates partial derivative, e.g. $s_{i,j}$ is equivalent to $\partial s_i/\partial x_j$, (ε, γ) represent longitudinal and shear strain and (k_1, k_2, k_3) are coefficients used to summarize, within a single formula, several displacement assumptions usually encountered in literature. Namely, by imposing $k_1 = k_2 = 0$, $k_3 = 1$ Eq. (2) returns the von Kármán kinematic model whereas, imposing $k_1 = k_2 = k_3 = 1$, it provides the Green–Lagrange strain model. Furthermore, two intermediate models can be considered by setting $k_1 = 1$ and $k_2 = 0$ or $k_1 = 0$ and $k_2 = 1$ (i.e. isolating the non linear contribute of one of the in plane displacement component derivatives of s_x or s_y).

By substituting Eq. (1) into Eq. (2), the strain components:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_L + \boldsymbol{\varepsilon}_{NL} \tag{3}$$

is obtained in terms of the generalized displacement $(u, v, w, \varphi_x, \varphi_y)$, where:

$$\boldsymbol{\varepsilon}_L = \left\{ \begin{array}{c} u_{,x} - z\varphi_{x,x} \\ v_{,y} - z\varphi_{y,y} \\ u_{,y} + v_{,x} - z(\varphi_{x,y} + \varphi_{y,x}) \\ w_{,x} - \varphi_x \\ w_{,y} - \varphi_y \end{array} \right\} \tag{4}$$

collects the linear components of the strain field, and:

$$\boldsymbol{\varepsilon}_{NL} = \frac{1}{2}k_1 \begin{bmatrix} (u_{,x} - z\varphi_{x,x})^2 \\ (u_{,y} - z\varphi_{x,y})^2 \\ 2(u_{,x} - z\varphi_{x,x})(u_{,y} - z\varphi_{x,y}) \\ 2\varphi_x(z\varphi_{x,x} - u_{,x}) \\ 2\varphi_x(z\varphi_{x,y} - u_{,y}) \end{bmatrix} + \frac{1}{2}k_2 \begin{bmatrix} (v_{,x} - z\varphi_{y,x})^2 \\ (v_{,y} - z\varphi_{y,y})^2 \\ 2(v_{,x} - z\varphi_{y,x})(v_{,y} - z\varphi_{y,y}) \\ 2\varphi_y(z\varphi_{y,x} - v_{,x}) \\ 2\varphi_y(z\varphi_{y,y} - v_{,y}) \end{bmatrix} + \frac{1}{2}k_3 \begin{bmatrix} w_{,x}^2 \\ w_{,y}^2 \\ 2w_{,x}w_{,y} \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

the remaining nonlinear terms contained in Eq. (2).

The equilibrium state is thus sought by invoking the principle of minimum of the strain energy.

The pre-critical kinematical configuration does not have out of plane displacement components $w = \varphi_x = \varphi_y = 0$, whereas the in-plane components $u = u_0$, $v = v_0$ generate the following pre-critical stress field $\boldsymbol{\sigma}^0$:

$$\begin{bmatrix} \sigma_x^0 \\ \sigma_y^0 \\ \tau_{xy}^0 \end{bmatrix} = \frac{1}{h}\begin{bmatrix} N_x^0 \\ N_y^0 \\ N_{xy}^0 \end{bmatrix} = \begin{bmatrix} \frac{E_x}{1-\nu_{xy}\nu_{yx}} & \frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ \frac{\nu_{yx}E_y}{1-\nu_{xy}\nu_{yx}} & \frac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{bmatrix} \tag{6}$$

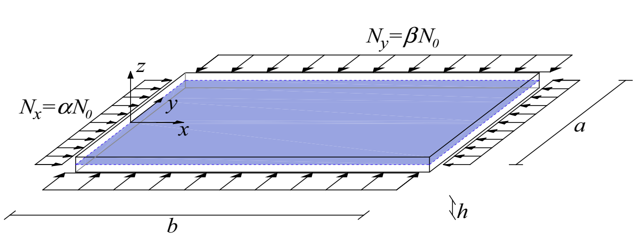

Fig. 1. Coordinate systems, geometry and applied in-plane loads for the generic *i*th plate.