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Two-dimensional strain-based interactive failure theory for multidirectional composite laminates

Soo-Yong Lee *, Jin-Ho Roh

School of Aerospace and Mechanical Engineering, Korea Aerospace University, 200-1 Hwajeon, Goyang, Gyeonggi, Republic of Korea

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ABSTRACT

A 2-D strain-based interactive failure theory is developed to predict the final failure of composite laminates subjected to multi-axial in-plane loading. The stiffness degradation of a laminate during loading is examined based on the individual failure modes of the maximum strain failure theory, and a piecewise linear incremental approach is employed to describe the nonlinear mechanical behavior of the laminate. In addition, an out-of-plane failure mode normal to the laminate is also investigated to more accurately predict the failure of multidirectional laminates. The theoretical results of the failure model presented are compared with the experimental data provided by the World-Wide Failure Exercise, and the accuracy of the model's predictive capabilities is investigated.

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1. Introduction

Failure prediction for fiber reinforced composites under complex loading is vitally important for efficient design in structural applications. Over the past several decades, a number of researchers have endeavored to more accurately predict the failure of multidirectional composite laminates and develop appropriate failure theories. However, a variety of failure theories have differences in terms of theoretical approach, methodology, and assumptions.

To investigate the differences and predictive capabilities of these failure theories developed for composite laminates, Soden and Hinton [1] launched the World-Wide Failure Exercise (WWFE). In this exercise, the participants were asked to submit papers containing both their own detailed theory and a comparison of their theoretical results with experimental data, according to the procedures provided by the organizers [2,3]. According to Ref. [4], a total of 11 groups took part in the exercise and presented 14 different theories [5–16]. Additional five failure theories [17–21] were presented since the publication of the first comparative study [22]. Comments on the individual failure theories in the WWFE are summarized in detail in Refs. [23,24]; the theories were evaluated according to their ability of predicting the experimental results for the response of a lamina, initial and final strength of multidirectional laminates and the deformation of laminates. Based on the failure theory employed in the WWFE study, several researches showed some intrinsic limitation on the theory, and suggested additional numerical schemes and experimental tests to improve the predictive performance of the failure of composite laminates [25–30].

Cuntze and Freund [19] developed a 3-D invariant-based failure theory with the so-called Failure Mode Concept (FMC) where the interaction between failure mechanisms is assumed to consider probabilistic effects. This theory has been recognized as one of the best theories currently available in the field [24]. However, for a good fit with experimental data, additional factors such as the rounding-off exponent and three free curve parameters derived from multidirectional test data would be required. Zinoviev et al. [16] developed a very simple and carefully structured non-interactive failure theory based on the maximum stress theory, considering the stiffness degradation and unloading of a failed lamina and fiber re-orientation during loading. The theory predicted very well overall and was evaluated as one of the best by Hinton et al. [23]. Bogetti et al. [18] employed a maximum strain failure criterion formulated in three dimensional forms and took into account lamina non-linear shear behavior and progressive failure. In the theory, the final failure of a composite laminate is assumed to occur, not by a certain criterion, but when the laminate loses sufficient stiffness such that it cannot carry any load without undergoing an arbitrarily excessive amount of deformation strain. Its theoretical results agreed well with experimental data and its performance was also highly ranked by Hinton et al. [24]. Liu and Tsai [14] employed a quadratic failure theory well known as the Tsai–Wu failure criterion for the WWFE; it too was highly ranked by Hinton et al. [24]. The theory analyzed the failure of composite laminates through the degradation of longitudinal compressive

* Corresponding author. Tel.: +82 2 300 0111.
E-mail address: leesy@kau.ac.kr (S.-Y. Lee).

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strength as well as the degradation of lamina stiffness due to matrix micro-cracking. However, the degradation of matrix micro-cracking, depending on the sign of a transverse normal strain of lamina stiffness, was ignored in the revised paper [31].

In this paper, a 2-D strain-based interactive failure theory is derived to predict the final failure of multidirectional composite laminates subjected to multi-axial in-plane loading. A piecewise linear incremental approach is adopted to describe nonlinear material behavior. The stiffness degradation of a failed ply (layer) under loading is examined by analyzing the individual failure modes of the maximum strain theory. In the case of multidirectional laminates under biaxial compression loading, delamination between their layers may occur and ultimately lead to local buckling failure. Therefore, an out-of-plane strain failure mode normal to the laminate is considered to investigate its effects on the failure of laminates. The accuracy of the failure model developed in this paper is verified and discussed by comparing its theoretical results with the experimental data of unidirectional and multidirectional laminates under biaxial loading.

2. Interactive failure theory

For the failure prediction of composite laminates under loading, most failure theories currently available have been developed and adequately modified based on the maximum stress theory, the maximum strain theory, and the quadratic failure theory known as the Tsai–Wu theory. However, these theories seem to agree well with the experimental data for unidirectional laminates, but not always for multidirectional laminates subjected to multi-axial loading.

In this paper, local and global coordinate systems mean the on-axis (principal material) and off-axis of a laminate and are denoted as 1–2–3 and x–y–z axes, respectively, in Fig. 1. The fiber orientation θ is measured from the x-axis to the 1-axis. Also, unbarred and barred notations are used to denote variables in the local and global coordinates, respectively. For a state of plane stress, the stress–strain constitutive relationship for a single orthotropic layer (ply) of a fiber reinforced composite laminate is expressed as

$$\sigma_i = Q_{ij}\varepsilon_j \quad i,\ j = 1,2,6 \tag{1}$$

$$Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}} \quad Q_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}} \tag{2}$$

$$Q_{12} = \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} \quad Q_{66} = G_{12} \tag{3}$$

where Q_{ij} is the reduced stiffness matrix of a layer. E_{11}, E_{22} and G_{12} represent longitudinal, transverse and in-plane shear moduli, respectively, and υ_{12} and υ_{21} are the major and minor Poisson's ratios. The constitutive equation for a laminate composed of n layers and subjected to in-plane loading $\mathbf{N}$ in Fig. 2 is written below

$$\mathbf{N} = \mathbf{A}\bar{\boldsymbol{\varepsilon}}^{\mathbf{o}} \quad \text{or} \quad \bar{\boldsymbol{\varepsilon}}^{\mathbf{o}} = \mathbf{A}^{-1}\mathbf{N} = \boldsymbol{a}\mathbf{N} \tag{4}$$

$$A_{ij} = \int_{-h/2}^{h/2} \overline{Q}_{ij}\, dz \tag{5}$$

The symbols of $\mathbf{A}$ and $\boldsymbol{a}$ represent the in-plane stiffness and compliance tensors of the laminate, respectively. The symbol of $\bar{\boldsymbol{\varepsilon}}^{\mathbf{o}}$ is the mid-plane strain of the laminate and its upper script **o** is dropped off hereafter, and $\overline{Q}_{ij}$ is the off-axis modulus of a layer.

Most composite laminates usually show nonlinear material behavior and their stiffness may be degraded by failure in a certain layer during loading. Therefore, in the present study, the piece-wise linear incremental approach is employed to consider the nonlinear behavior and the degradation of composite laminates. In this approach, the individual ply stresses and strains are computed in each step during the incremental loading $\Delta\mathbf{N}$ as follows.

$$\Delta\bar{\boldsymbol{\varepsilon}} = \boldsymbol{a}\Delta\mathbf{N} \tag{6}$$

$$\Delta\boldsymbol{\varepsilon} = \mathbf{T}_{\varepsilon}\Delta\bar{\boldsymbol{\varepsilon}} \tag{7}$$

$$\Delta\boldsymbol{\sigma} = \mathbf{Q}\Delta\boldsymbol{\varepsilon} \tag{8}$$

The symbol of $\mathbf{T}_{\varepsilon}$ represents the strain transformation tensor. After each load step, the local stresses and strains and the loads are updated as follows.

$$\boldsymbol{\sigma}^{\mathbf{i+1}} = \boldsymbol{\sigma}^{\mathbf{i}} + \Delta\boldsymbol{\sigma} \tag{9}$$

$$\boldsymbol{\varepsilon}^{\mathbf{i+1}} = \boldsymbol{\varepsilon}^{\mathbf{i}} + \Delta\boldsymbol{\varepsilon} \tag{10}$$

$$\mathbf{N}^{\mathbf{i+1}} = \mathbf{N}^{\mathbf{i}} + \Delta\mathbf{N} \tag{11}$$

First of all, the stiffness degradation of a multidirectional laminate subjected to multi-axial loading is examined based on individual failure modes of the maximum strain failure theory which is expressed as follows.

Longitudinal failure mode:

$$G_{11}\varepsilon_1^2 + G_1\varepsilon_1 < 1 \quad (\text{or } -\varepsilon_{1c} < \varepsilon_1 < \varepsilon_{1t}) \tag{12}$$

$$G_{11} = \frac{1}{\varepsilon_{1t}\varepsilon_{1c}}, \quad G_1 = \frac{1}{\varepsilon_{1t}} - \frac{1}{\varepsilon_{1c}} \tag{13}$$

Transverse failure mode:

$$G_{22}\varepsilon_2^2 + G_2\varepsilon_2 < 1 \quad (\text{or } -\varepsilon_{2c} < \varepsilon_2 < \varepsilon_{2t}) \tag{14}$$

$$G_{22} = \frac{1}{\varepsilon_{2t}\varepsilon_{2c}}, \quad G_2 = \frac{1}{\varepsilon_{2t}} - \frac{1}{\varepsilon_{2c}} \tag{15}$$

In-plane shear failure mode:

$$G_{66}\varepsilon_6^2 < 1 \quad (\text{or } |\varepsilon_6| < \varepsilon_s) \tag{16}$$

$$G_{66} = \frac{1}{\varepsilon_s^2} \tag{17}$$

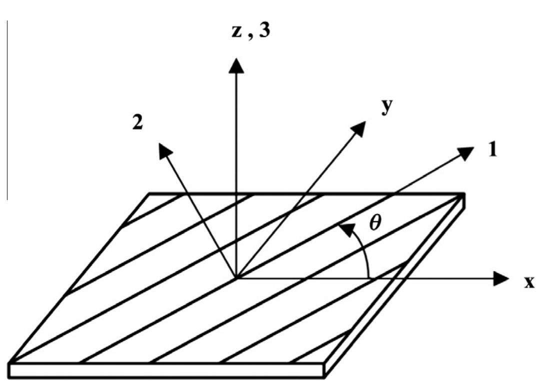

Fig. 1. Local and global coordinate systems of a composite laminate.

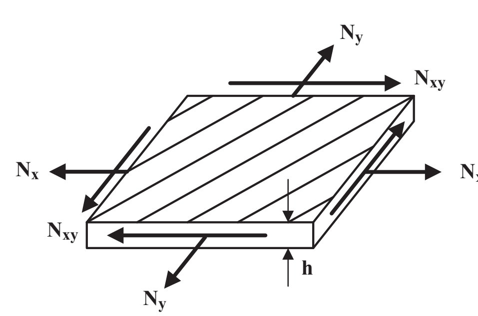

Fig. 2. In-plane loading of a composite laminate.

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