Composites: Part B 69 (2015) 120–126

Contents lists available at ScienceDirect

Composites: Part B

journal homepage: www.elsevier.com/locate/compositesb

Buckling of elastoplastic functionally graded cylindrical shells under combined compression and pressure

Yongqiang Zhang, Huaiwei Huang *, Qiang Han

School of Civil Engineering and Transportation, South China University of Technology, Guangzhou, Guangdong 510640, PR China

ARTICLE INFO

Article history:
Received 23 April 2014
Received in revised form 10 August 2014
Accepted 23 September 2014
Available online 2 October 2014

Keywords:
A. Metal–matrix composites (MMCs)
B. Buckling
B. Plastic deformation

ABSTRACT

Buckling behaviors of elastoplastic functionally graded material cylindrical shells under combined axial compression and external pressure are investigated with classical shell theory. The material properties vary smoothly through the thickness, and a multi-linear hardening elastoplasticity is used in the analysis. By extending TTO model of functionally graded materials into J_2 deformation theory, the elastoplastic constitutive relation of FGMs is founded. The buckling governing equations are solved by Galerkin method, and the expression of the critical condition under combine in-plane loads is given. Numerical results are given through an iterative procedure between the prebuckling state and the critical condition. Numerical results give the interactive curves of the stability regions and the exact elastoplastic interface of the materials. It is interesting to find that, material plastic flow is of significant effects on the stability region, and the effects of the constituent distribution and the elastoplastic material properties are discussed as well.

1. Introduction

Functionally graded materials (FGMs) are ceramic/metallic composites, with the constituents varying smoothly through the material thickness which enables the continually changing material properties as well [1]. This would effectively avoid stress concentration seen in traditional laminate or fiber-reinforced composites. Currently, FGMs have been widely applied in aerospace and nuclear industries [2].

As one of the fundamental issues of structural stability, buckling of FGM plates and shells have received considerable concern since 1997, when Feldman [3] investigated buckling of FGM rectangular plates. Then, much interest had been attracted to elastic buckling issues [4–6]. The research topics covered a wild range of buckling behaviors of FGM plates and shells, geometrically linear or nonlinear, static or transient and etc..

Recently, some attention was attracted to elastoplastic FGMs. The material properties of FGMs can be depicted by a homogenization rule of mixture, named TTO model, initially proposed by Tamura et al. [7] for metal alloy. Bocciarelli et al. [8] extended TTO model to J_2 flow theory with isotropic hardening to describe the elastoplastic behaviors of FGMs and he pointed out that TTO model is an effective homogenization rule governing the transition from Hencky-Huber-Mises (HHM) model, typical of metals, toward a Drucker-Prager constitutive model which is more suitable to describe the mechanical response of ceramics. Meanwhile, an inverse analysis procedure, based on indentation tests, was developed to identify the stress transfer parameters in TTO model [9,10].

Currently, researches on mechanical performances of elastoplastic FGM structures primly focused on thermal responses [11,12] and cracking resistance [13,14] of FGM structures. Few literature reported elastoplastic buckling of FGM plates and shells. Generally, in ceramic/metallic FGMs, ductile metallic constituents may initiate severe plastic deformation when stresses rise. As general knows, material plasticity would greatly reduce the buckling critical load of homogeneous shells [15]. However, for FGM cylindrical shells, it is still interesting to explore this effect. In this paper, buckling behaviors of elastoplastic FGM cylindrical shells under combined axial compression and external pressure are investigated with classical shell theory and J_2 deformation theory.

2. Material constitutive relation

Generally, the constituent distribution of FGMs submits the power law ruler [6]

$$V_c = (0.5 + z/h)^k,\ V_c + V_m = 1 \quad (1)$$

where k is the power law index, which is a critical parameter of constituent distribution. V denotes the volume fraction. The

* Corresponding author. Tel.: +86 1366 030 2875.
E-mail address: cthwhuang@scut.edu.cn (H. Huang).

http://dx.doi.org/10.1016/j.compositesb.2014.09.024
1359-8368/

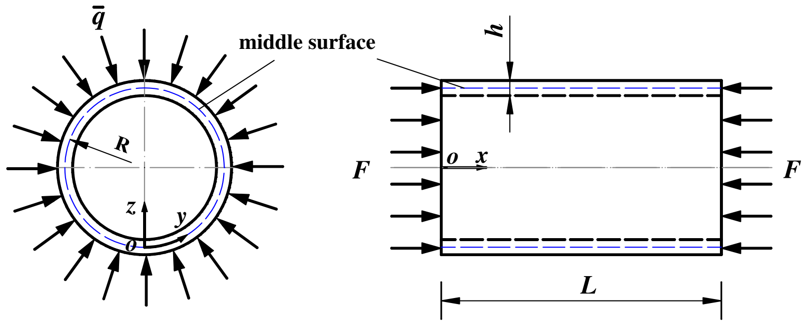

Fig. 1. Geometry and the coordinate system of combine-loaded FGM cylindrical shells.

subscripts c, m respectively correspond to the ceramic and metallic constituents.

In FGMs, ceramic constituents are usually brittle materials of relatively higher elastic modulus and strength than those of metallic constituents, which are typically ductile materials. According to the TTO model, the ceramic constituents are assumed to be elastic. Material flow of FGMs mainly arouse by plastic flow of the metallic constituent. Thus, multi-linear hardening elastoplastic material properties of FGMs [9] can be defined as

$$
\begin{aligned}
E &= \left(\frac{q+E_c}{q+E_m}E_m V_m + E_c V_c\right) \Big/ \left(\frac{q+E_c}{q+E_m}V_m + V_c\right) \\
\sigma_Y &= \sigma_{Ym}\left(V_m + \frac{q+E_m}{q+E_c}\frac{E_c}{E_m}V_c\right) \\
H &= \left(\frac{q+E_c}{q+H_m}H_m V_m + E_c V_c\right) \Big/ \left(\frac{q+E_c}{q+H_m}V_m + E_c\right)
\end{aligned}
\tag{2}
$$

where $E(z)$ is elastic modulus, $\sigma_Y(z)$ yield limit, and $H(z)$ the tangent modulus. $q = \tilde{q}E_c$ is the ratio of stress to strain transfer. $\tilde{q}$ is the stress transfer parameter, and $\tilde{q} \geq 0$. It should be note that $\tilde{q} = 0$ represents the FGMs flow plastically once the metallic constituent reach the yield limit.

The most popular elastoplastic constitutive relations of homogeneous materials are J_2 flow theory and J_2 deformation theory. According to Mao and Lu [15], for axial compressed homogeneous cylindrical shells, J_2 deformation theory acutely predicted the plastic buckling critical load of experiments, while J_2 flow theory was of enormous deviation. This is the well-known plastic buckling paradox which is still open to question in modern structural stability theory [16]. Thus, the following analysis would be presented by using J_2 deformation theory, the constitutive relation of FGMs can be given as

$$
\varepsilon_{ij} = \frac{3}{2E_s}\sigma_{ij} + \left(\frac{1}{K} - \frac{3}{2E_s}\right)\delta_{ij}\sigma_m \tag{3}
$$

in which, the subscript i, j represent x, y, z. The secant modulus in complex stress state $E_s = 3EE_s^0/[3E - (1-2\nu)E_s^0], K = E/(1-2\nu)$, E_s^0 is the secant modulus in the uniaxial tension experiment. The mean stress $\sigma_m = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$. δ_{ij} is unit matrix. The incremental form of Eq. (3) reads

$$
d\varepsilon_{ij} = \frac{3}{2E_s}d\sigma_{ij} + \left(\frac{1}{K} - \frac{3}{2E_s}\right)\delta_{ij}d\sigma_m + \frac{3}{4J_2\phi}S_{ij}S_{ld}d\sigma_{ld} \tag{4}
$$

in which $\phi = E_t E_s/(E_s - E_t)$. The tangent modulus in complex stress state $E_t = 3EH/[3E - (1 - 2\nu)H]$. S_{ij} is the tensor of stress deviator. $J_2 = S_{ij}S_{ij}/2$. According to classic shell theory, the stresses out of the plane $\sigma_{xz}, \sigma_{yz}, \sigma_{zz}$ can be neglected. Accordingly, Eq. (4) is reduced as

$$
\begin{aligned}
d\varepsilon_{xx} &= \left(\frac{1}{3K} + \frac{1}{E_s}\right)d\sigma_{xx} + \left(\frac{1}{3K} - \frac{1}{2E_s}\right)d\sigma_{yy} + \frac{3}{4J_2\phi}(\sigma_{xx} - \sigma_m)d\tilde{\sigma} \\
d\varepsilon_{yy} &= \left(\frac{1}{3K} - \frac{1}{2E_s}\right)d\sigma_{xx} + \left(\frac{1}{3K} + \frac{1}{E_s}\right)d\sigma_{yy} + \frac{3}{4J_2\phi}(\sigma_{yy} - \sigma_m)d\tilde{\sigma} \\
d\varepsilon_{zz} &= \left(\frac{1}{3K} - \frac{1}{2E_s}\right)(d\sigma_{yy} + d\sigma_{xx}) - \frac{3}{4J_2\phi}\sigma_m d\tilde{\sigma} \\
d\varepsilon_{xy} &= d\varepsilon_{yx} = \frac{3}{E_s}d\sigma_{xy} + \frac{3}{2J_2\phi}\sigma_{xy}d\tilde{\sigma}, d\varepsilon_{xz} = d\varepsilon_{zx} = d\varepsilon_{yz} = d\varepsilon_{zy} = 0 \\
J_2 &= \frac{1}{3}(\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3\sigma_{xy}^2), \sigma_m = \frac{1}{3}(\sigma_{xx} + \sigma_{yy}), \\
d\tilde{\sigma} &= [(\sigma_{xx} - \sigma_m)d\sigma_{xx} + (\sigma_{yy} - \sigma_m)d\sigma_{yy} + 2\sigma_{xy}d\sigma_{xy}]
\end{aligned}
\tag{5}
$$

3. Formulation

Assume FGM cylindrical shells are subjected to uniform axial compression force F and external pressure $\bar{q}$ as shown in Fig. 1. The geometry can be demonstrated by thickness h, length L, and mean radius R, and the coordinate system is placed on the middle surface, with the origin o at its left end and the coordinate axes x, y, and z in the axial, circumferential, and the inward normal directions respectively.

According to the nonlinear von Kárman strain–displacement relations of cylindrical shells, the incremental strain components on the middle plane of the shells are

$$
d\varepsilon_{xx}^0 = du_{,x} + \frac{1}{2}dw_{,x}^2, \quad d\varepsilon_{yy}^0 = d\nu_{,y} - \frac{dw}{R} + \frac{1}{2}dw_{,y}^2, \quad d\varepsilon_{xy}^0 = du_{,y} + d\nu_{,x} + dw_{,x}dw_{,y} \tag{6}
$$

where d denotes the increment. u, ν, w are displacements along x, y, z, and the subscript comma denotes partial derivative.

The incremental strain components of cylindrical shells are

$$
d\varepsilon = d\varepsilon^0 + z dK \tag{7}
$$

where $d\varepsilon = \left[d\varepsilon_{xx} \quad d\varepsilon_{yy} \quad d\varepsilon_{xy}\right]^T, dK = \left[dK_{xx} \quad dK_{yy} \quad dK_{xy}\right]^T, d\varepsilon^0 = \left[d\varepsilon_{xx}^0 \quad d\varepsilon_{yy}^0 \quad d\varepsilon_{xy}^0\right]^T$ and the curvature components are

$$
dK_{xx} = -dw_{,xx}, \quad dK_{yy} = -dw_{,yy}, \quad dK_{xy} = -2dw_{,xy} \tag{8}
$$

The incremental stress components can be given by rewriting Eq. (4) as

$$
d\sigma = A d\varepsilon \tag{9}
$$

where $d\sigma = [d\sigma_{xx} \quad d\sigma_{yy} \quad d\sigma_{xy}]^T$ and the matrix $A = [a_{ij}]$, $(i, j = 1, 2, 3)$. It should be noted that a_{ij} are stress-dependent material parameters demonstrating anisotropy of elastoplastic FGMs. For cylindrical shells, the incremental internal force and moment components