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Incremental mean-fields micromechanics scheme for non-linear response of ductile damaged composite materials

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ABSTRACT

This work is concerned with the modeling of ductile damage behavior in composite materials by the means of the Incremental Micromechanics Scheme (IMS) as Mean-Fields Homogenization (MFH) technique. Indeed, IMS is known for its capability to overcome the well-known accuracy restrictions of the Mori–Tanaka (MT) and Self-Consistent (SC) schemes when a high volume fraction of heterogeneities or/and a high contrast between phases properties is reached. This micromechanics formalism is based on the Eshelby's inclusion concept. The kinematic equation of Dederichs and Zeller (1973) is used as formal solution of the heterogeneous material problem. The nonlinear behavior of the composite is addressed in a general framework based on the kinematic hardening of Lemaître–Chaboche's ductile damage model. Thus a classical J_2 plasticity that accounts for the damage evolution within the microstructure is implemented. The time discretization of all rate relations is solved through a generalized mid-point rule that yields to an anisotropic consistent (algorithmic) tangent modulus. To avoid a stiffer macroscopic stress–strain response, an isotropization procedure is adopted during the computation of the Eshelby tensor involved in the IMS modeling. From a computational aspect, the non linear response of the composite is obtained through two interdependent loops: inner and outer. In the inner loop, the IMS determines the global strain concentration tensor that is passed to the outer loop. Then, the macroscopic stress–strain response is derived using an iterative algorithm based on the Hill-type incremental formulation. Numerical results are obtained considering several heterogeneous materials such as Metal Matrix Composites (MMCs) as well as Carbon fibers reinforced Epoxy Matrix Composites. The model's predictions are compared in most of the cases, with experimental data and predictions obtained from MT-based modeling in the open literature.

1. Introduction

In several technological domains, the design of composite materials retains serious attentions on the safety, economy and durability of material's constituents in terms of damage initiation. As damage characterizes a local approach of the failure, its understanding and physical description present a major challenge in various engineering-related disciplines [1]. For ductile materials, the damage's onset begins at microscale by the nucleation, growth and coalescence of micro-cavities, for instance in Metal Matrix Composites (MMCs) [2–5]. These occurred phenomena within the microstructure require the use of robust tools to account for their effects at the macroscopic level. Thus the micromechanics which establishes relationship between continuum properties of a material and its microstructure, offers an ideal framework to treat that issue. The micro–macro transition is often set up through homogenization tools among which are the MFH approaches. For multi-scale analysis, MFH approaches constitute a good compromise between the prediction's accuracy and the computational cost through either analytically and/or numerically derivations from the constituents' properties [6].

Since relevant works of Eshelby [7] about the ellipsoidal inclusion concept, MFH approaches have gained noteworthy progress and have been intensively used for the evaluation of the effective elastic properties of heterogeneous composites [8]. Indeed, from earlier works provided by Voigt and Reuss on the micromechanics bounds, more accurate models have been developed accounting for the morphological and topological textures of material constituents. One can recall first SC schemes applied to heterogeneous materials by Hershey [9] and Kroner [10]. Another used scheme is the M–T scheme developed by Mori and Tanaka [11]. Its

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derivation is based on linear resolution of the kinematic integral equation of Dederichs and Zeller [12] and focuses particular interests because of its good prediction of heterogeneous effective mechanical properties at low and moderate volume fractions of reinforcements.

For non-linear behavior, the extension of MFH approaches to account for such a response has been widely studied. These techniques are based on Hill-type linearization [13] which provides an incremental resolution of the non linear stress–strain problem. Within this framework, several MFH models are used and discussed from a broad literature. One can call up earlier works of Hutchinson [14] using the SC scheme and that of Tandon and Weng [15] using the M–T scheme or Double Inclusion (DI) derivations provided by Nemat-Nasser and Hori [16]. More recently, the M–T strategy is used by Doghri and Ouaar [17], Pierard and Doghri [18] to derive the non-linear behavior of elasto-plastic composites. Wu et al. [6] use the M–T modeling to study the non-linear behavior of ductile damage matrix composites with an emphasis on the strain/damage localization due to lost of ellipticity. Others approaches like the so-called affine method and secant method are also used to tackle non-linear homogenization. A comprehensive overview of these approach can be found in [19,20]. The above-mentioned models have been used with success to predict the non-linear behavior of heterogeneous materials with low or moderate reinforcement volume fractions. To deal, for instance, with long fibers reinforced composites as reported in [21,22], where experiments are done on E-glass fibers/nylon-6 and Long Glass Fibers (LGF)/polymer PA6 with high volume fractions of 66.5, 68.8, 71.1 and 73.3 vol.%, it turns out to set up an alternative approach that overcomes the well-known accuracy restrictions of classical MFH approaches. Therefore, the Incremental Micromechanics Scheme (IMS) has been developed by Vieville [23] and Vieville and Lipinski [24] for that issue. IMS has demonstrated its performance when a high volume fraction or/and a high contrast between phases properties is reached. The composite is built in several iterative steps by a gradual addition of infinitesimal quantities of reinforcements. Obviously, more CPU cost time can be highlighted to obtain the final homogenized properties. However, IMS is valid for treating any kind of anisotropic composite materials and permits to go beyond accurate limitations of others MFH schemes. It is based on the idea of the Differential Scheme (DS) and offers the great advantage that one has not to resolve any tensorial differential equations. This approach has been successfully used to predict elastic properties of heterogeneous materials [25,26] and more recently by Azoti et al. [27] in a modeling that account for the elasto-plastic behavior.

In this work, a formulation of applying the IMS to composites undergoing ductile damage deformation is proposed. The micromechanics framework is based on the kinematic integral equation of Dederichs and Zeller [12] as formal solution for the heterogeneous material's problem. Based on works of Vieville et al. [28], a formulation of IMS derived from the dilute concentration procedure is applied. At each iterative step, IMS deals with a homogenization technique that considers the reference medium as the equivalent material at the previous step. For each material's constituent, the damage behavior is introduced through the so-called effective stress $\hat{\boldsymbol{\sigma}}$. The damage variable D follows Lemaitre and Chaboche's law in which the accumulated plastic strain is derived from the classical J_2 flow rule with a kinematic hardening. This enables to properly account for the Bauschinger effect when dealing with cyclic loadings. Following works of Doghri [29], the consistent (algorithmic) modulus is obtained for constituents. The well-known damage localization problem is not addressed herein since critical level of loadings corresponding to the lost of uniqueness are not reached in applications. To this end, at each time increment, the positive-definite properties of the algorithmic modulus is checked by computing its eigenvalues. Once, eigenvalues are negative, the IMS homogenization process leading to the effective tangent stiffness tensor is stopped. Also, during each iterative step of the IMS modeling, the Eshelby tensor involved in the global strain concentration is computed by an isotropization of the matrix phase. This isotropization discussed in Doghri and Ouaar [17] and Chaboche et al. [30] is essential to avoid stiff macro stress–strain responses.

In what follows, Section 2 establishes the general framework of a MFH by deriving the global strain concentration tensor $\mathbf{A}^I$. In the Section 3, the classical J_2 flow theory is recalled. In this section, the procedure for obtaining the ductile damage's internal variables is shown through the return mapping algorithm. The consistent (algorithmic) tangent operators are obtained and passed as information to the IMS modeling presented in Section 4. The capability of IMS to solve a ductile damage problem is shown in the Sections 5 and 6 through an iterative algorithm based on two (2) interdependent loop. The numerical results, on MMCs as well as epoxy based matrix composites in Section 7, are compared with experimental data and others solutions available from the open literature.

2. Background on MFH approaches

Most of materials are homogeneous at the macro-scale. In order to capture fine details, one has to step down inside the microstructure that appears heterogeneous. A Representative Volume Element (RVE), as suggested by Hill [31], Kroner [32,33] and Willis [34], can be defined and on which admissible static or kinematic loads can be applied (boundary value problem). The RVE is assumed to be in equilibrium and its overall strain is compatible. The body forces and inertia term are absents. These general considerations are restricted to the case of a linear constitutive laws with small transformations. The micromechanics scale transition consists firstly, in the localization of the macroscopic strain tensor $\mathbf{E}$ by the introduction of a fourth order global strain concentration tensor $\mathbf{A}(r)$ and secondly, in the homogenization process, which uses averaging techniques to approximate the macroscopic behavior. Note that $\mathbf{A}(r)$ remains the unknown parameter that contains all microstructural informations. The effective properties of the RVE are given by:

$$\mathbf{C}^{eff} = \frac{1}{V}\int_V \mathbf{c}(r) : \mathbf{A}(r) dV \tag{1}$$

where $\mathbf{c}(r)$ denotes the local uniform modulus and V is the volume of the RVE. The operator ":" states for tensorial product contracted over two indices. The global strain concentration tensor $\mathbf{A}(r)$ links the local strain $\epsilon(r)$ and the macroscopic strain $\mathbf{E}$ as follow:

$$\epsilon(r) = \mathbf{A}(r) : \mathbf{E} \tag{2}$$

The local uniform modulus is split into a reference part $\mathbf{c}^R$ and a fluctuation part $\delta\mathbf{c}$ such as $\mathbf{c}(r) = \mathbf{c}^R(r) + \delta\mathbf{c}(r)$. This latter relation can be substituted within the equilibrium equation $\boldsymbol{\sigma}_{ij,j} = 0$ and therefore enables the derivation of the kinematic integral equation of Dederichs and Zeller [12]. In terms of strain fields, the kinematic integral equation arises:

$$\epsilon(r) = \mathbf{E}^R - \int_V \boldsymbol{\Gamma}(r - r') : \delta\mathbf{c}(r') : \epsilon(r')dV' \tag{3}$$

where $\mathbf{E}^R$ is the homogeneous strain field within the reference medium and $\boldsymbol{\Gamma}(r - r')$ is the modified Green tensor. Based on the kinematic integral Eq. (3) and the Eshelby's inclusion concept [7] for ellipsoidal inclusions, Vieville et al. [28] derived an expression of the global strain concentration tensor $\mathbf{A}^I(r)$ inside an Ith inclusion through an iterative procedure such as: