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Luneburg-lens-like structural Pauli attractive core of the nuclear force at short distances

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Abstract

A recent paper Ohkubo (2017) [1] found that the measured $^1S_0$ phase shifts can be reproduced using a deeply attractive nucleon–nucleon potential. We find that the deuteron would decay strongly via pion emission to the deeply bound state arising in this potential. Therefore the success of a deeply attractive potential in describing phase shifts must be regarded only as an interesting curiosity.

Keywords: Deeply attractive nucleon–nucleon potential; Pion emission; Deuteron stability

1. Introduction

A recent paper [1] finds a nuclear force with an attractive potential at short distances that reproduces the experimental $^1S_0$ phase shifts well. Such a potential can be motivated by early quark-model ideas [2], but later work [3] showed that quark model ideas lead to short distance repulsion between nucleons. Here we show that the deep attraction causes a deeply bound state to exist, with the drastic consequence that the deuteron would not be stable.

The $^1S_0$ potential $V(r)$ of [1] is given by

$$V(r) = -5\,e^{-(r/2.5)^2} - 270\,e^{-(r/0.942)^2} - 1850\,e^{-(r/0.447)^2}$$
$$\equiv \sum_{n=1}^{3} V_n e^{-r^2/r_n^2}. \tag{1}$$

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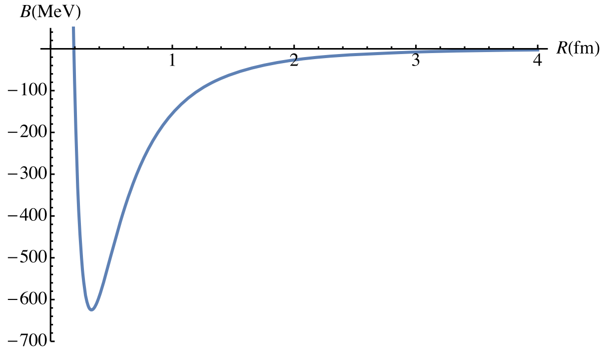

Fig. 1. (Color online.) The expectation value of the Hamiltonian, $B(R)$.

The strength parameters of V are given in units of MeV, and range parameters are in units of fm. This purely attractive potential has a depth of 2125 MeV at $r = 0$ and a half-width r_0 of about 0.4 fm. The corresponding uncertainty principle estimate of the kinetic energy, $\hbar^2/(Mr_0^2)$, with M as the nucleon mass, is 259 MeV, so that the quickest look at this potential leads to the conclusion that the existence of a deeply bound state is an immediate consequence of using Eq. (1).

The easiest analytic way to show that a bound state must exist is to use the variational principle. The single-parameter trial wave function $u(r)$ used here takes the form:

$$u(r) = \frac{2re^{-\frac{r^2}{2R^2}}}{\sqrt[4]{\pi}\sqrt{R^3}}, \tag{2}$$

with the normalization $\int_0^\infty dr\, u^2(r) = 1$. If the expectation value of the Hamiltonian, H, within this (or any) wave function is less than zero, the potential must yield a bound state. The expectation value of the H, defined as $B(R)$ is given by

$$B(R) = \frac{3\hbar}{2MR^2} + \sum_{n=1}^{3} V_n \frac{1}{(1+\frac{R^2}{r_n^2})^{3/2}}, \tag{3}$$

with the first, positive term arising from the kinetic energy much smaller than the negative potential energy terms. This can be seen immediately using only the $V_3 = -1850$ MeV term of Eq. (3). With $R = r_3$ the V_3 term is $V_3/(2\sqrt{2}) = -650$ MeV, while the kinetic energy term is about 390 MeV. Fig. 1 shows that $\langle H \rangle \equiv B(R)$ bottoms out at about -620 MeV. Thus there must be a bound state, and its binding energy must be greater than or equal to 620 MeV. Numerical solution of the Schrödinger equation yields a binding energy of about 640 MeV [1].

In Ref. [1], this state is denoted as "unphysical" and "Pauli forbidden". However, the Pauli principle does not forbid a $^1S_0$ bound state. For example 6 quarks each in the lowest orbital of the MIT bag model form a bound state in that channel if gluon exchange effects are neglected [4] and such states could play an important role in nucleon–nucleon scattering [3,5].

A deeply bound $^1S_0$ state has never been found and our very existence shows that this bound state cannot be real. This is because the deuteron would decay strongly to this bound state by the emission of a pion.