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Generalized entropy formalism and a new holographic dark energy model

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ABSTRACT

Recently, the Rényi and Tsallis generalized entropies have extensively been used in order to study various cosmological and gravitational setups. Here, using a special type of generalized entropy, a generalization of both the Rényi and Tsallis entropy, together with holographic principle, we build a new model for holographic dark energy. Thereinafter, considering a flat FRW universe, filled by a pressureless component and the new obtained dark energy model, the evolution of cosmos has been investigated showing satisfactory results and behavior. In our model, the Hubble horizon plays the role of IR cutoff, and there is no mutual interaction between the cosmos components. Our results indicate that the generalized entropy formalism may open a new window to become more familiar with the nature of spacetime and its properties.

1. Introduction

In standard cosmology, based on general relativity, one way to describe the current accelerating universe is to consider an unknown energy–momentum source called dark energy [1–3]. From thermodynamic point of view, dark energy candidates and horizon entropy can be affected by each other [4–8]. Recently, due to the unknown nature of spacetime, the long-range nature of gravity, and also motivated by the fact that the Bekenstein–Hawking entropy is a non-extensive entropy measure [9–13], the Rényi and Tsallis generalized entropies [9,10,13–20] have been attributed to horizons to study various cosmological and gravitational phenomena [11–13,21–50]. The successes of these attempts in modelling the current accelerating cosmos [37–48] encourage and motivate us to study the cosmos evolution in various generalized entropy setups which may help us to become familiar with the probable non-extensive features of spacetime, and thus its origin [38].

Based on spacetime thermodynamics, the apparent horizon of FRW universe is a proper causal boundary [51–53], meaning that the thermodynamics laws are satisfied on this boundary [54,55]. Moreover, WMAP data indicates a flat FRW universe [1], a universe for which apparent horizon is equal to the Hubble horizon. Thus, proper models of dark energy should be in agreement with the Hubble horizon in flat FRW background.

Following the Cohen et al.'s hypothesis on the mutual relation between the UV cutoff and the entropy of system [56], a new class of dark energy models have been proposed, called holographic dark energy (HDE) [57–66]. In flat FRW universe, the original model of HDE (OHDE) is constructed by attributing the Bekenstein–Hawking entropy to the cosmos horizon and also considering the Hubble horizon as its IR cutoff [57–62]. Although the density parameter of OHDE shows an admissible behavior from itself, its energy density scales with H^2 meaning that it behaves as dark matter during the cosmos evolution [60,61], and in fact, OHDE is not in harmony with the Hubble radius [60,61]. Besides, it is not always stable whenever it is dominant in cosmos and controls its expansion rate [62]. Due to such weaknesses of OHDE, various attempts have been made to modify this model [65,66].

Although various entropies have been used to get modified HDE [65,66], none of them consider the generalized entropy formalism to build a HDE model. As we have previously mentioned, the Rényi

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and Tsallis generalized entropies generate suitable models for the current universe, and thus, we are going to use such formalism to build a new model for HDE in flat FRW by considering the Hubble radius as its IR cutoff. Here, we use a special generalized entropy, a generalization of both the Rényi and Tsallis entropies, to build our model. In fact, our final aim of introducing this new holographic model is to show that the probable non-additive and non-extensive aspects of spacetime have theoretically enough potential to accelerate the universe in a consistent way with observations.

The paper is organized as follows. In the next section, after reviewing some generalized entropy formalisms, we introduce our model of HDE. In continue, we consider a non-interacting universe, for which there is no mutual interaction between the cosmos components, and study the evolution of system in Sec. 3. A summary on the present work is also presented in the last section. The unit of $c = \hbar = G = k_B = 1$, where k_B denotes the Boltzmann constant, has also been used in this paper.

2. Horizon entropy in generalized entropy formalism and holographic dark energy

Consider a system including W states, in which P_i is the probability of achieving the ith state satisfying the $\sum_{i=1}^{W} P_i = 1$ condition. In this manner, Shannon's entropy can be employed to build ordinary statistical mechanics and its corresponding thermodynamics in which additivity and extensivity are the backbone of all results. Some systems, such as those including long range interactions, do not necessarily preserve the additivity and extensivity properties [9,10,14–20]. These are generally the systems described better by a power law distribution of probabilities, namely P_i^Q where Q is a real parameter [67], instead of the ordinary P_i distribution meaning that other entropy measures are needed to describe these systems [15,16,67–69].

Rényi ($\mathcal{S}$) and Tsallis (S_T) entropies are two well-known of one-parameter generalized entropy defined as [14–16]

$$\mathcal{S} = \frac{1}{\delta} \ln \sum_{i=1}^{W} P_i^{1-\delta}, \tag{1}$$

$$S_T = \frac{1}{\delta} \sum_{i=1}^{W} \left(P_i^{1-\delta} - P_i \right),$$

where $\delta \equiv 1 - Q$. Combining the above one-parametric entropy measures with each other, we can find their mutual relation [14–16,37,38]

$$\mathcal{S} = \frac{1}{\delta} \ln(1 + \delta S_T). \tag{2}$$

There is also another generalized entropy measure, introduced by Sharma and Mittal [68,69], indeed a two-parametric entropy defined as [67–72]

$$S_{SM} = \frac{1}{1-r}\left(\left(\sum_{i=1}^{W} P_i^{1-\delta}\right)^{\frac{1-r}{\delta}} - 1\right), \tag{3}$$

where r is a new free parameter. Some basic properties of this entropy are addressed in Refs. [67–72] which show its compatibility with various systems and indicate that it is a generalization of both the Rényi and Tsallis entropy. In fact, we can see that the Rényi and Tsallis entropies are recovered at the appropriate limits of $r \to 1$ and $r \to 1 - \delta = Q$, respectively [67–72]. Using Eqs. (1) and (3), one can easily reach

$$S_{SM} = \frac{1}{R}\left((1 + \delta S_T)^{\frac{R}{\delta}} - 1\right), \tag{4}$$

where $R \equiv 1 - r$.

As we mentioned, systems including the long-range interactions are better described by generalized entropies based on the power law distributions of probability [18–20,67]. Gravity is also a long-range interaction which motivates physicist to use the Rényi and Tsallis generalized entropies in order to study the gravitational and cosmological systems [11–13,21–50]. Since S_{SM} is the generalized form of both $\mathcal{S}$ and S_T [67–72], we use S_{SM} to build a new HDE.

It has recently been argued that the Bekenstein–Hawking is a proper candidate for the Tsallis entropy [11–13,17,32–34,37–39] allowing us to replace S_T with S_B in the above equation which leads to

$$S_{SM} = \frac{1}{R}\left((1 + \frac{\delta A}{4})^{\frac{R}{\delta}} - 1\right), \tag{5}$$

for the Sharma–Mittal entropy. In order to obtain this result, we also used $S_B = \frac{A}{4}$, where A is the horizon area. For example, the Bekenstein–Hawking entropy is obtained by using the Tsallis formalism in order to calculate the entropy of black holes in loop quantum gravity [13]. Thus, bearing Eq. (4) in mind, we can say that Eq. (5) is in fact the Sharma–Mittal entropy content of system.

Sharma–Mitall Holographic Dark Energy (SMHDE)

Based on the holographic principle, the IR (L) and UV (Λ) cutoffs are in relation with the system horizon (S) as [61,63]

$$\Lambda^4 \propto \frac{S}{L^4}. \tag{6}$$

In HDE hypothesis, the zero-point energy density (ρ_Λ) corresponding to the cut-off Λ ($\rho_\Lambda \sim \Lambda^4$), plays the role of the energy density of dark energy (ρ_D) meaning that we have $\rho_D \sim \Lambda^4$ [61,63]. Now, considering the Hubble radius as the IR cutoff leading to $L \equiv H^{-1} = \sqrt{\frac{A}{4\pi}}$, and by using Eqs. (5) and (6), we reach at

$$\rho_D = \frac{3C^2 H^4}{8\pi R}[(1 + \frac{\delta\pi}{H^2})^{\frac{R}{\delta}} - 1]. \tag{7}$$

Here, C^2 is the unknown free parameter as usual, for the energy density of SMHDE. The original HDE model is also obtainable at the appropriate limit of $R \to \delta$. Here, we consider a setup in which there is no interaction between various components of cosmos, meaning that SMHDE obeys ordinary conservation law, and thus

$$p_D = -(\frac{\dot{\rho}_D}{3H} + \rho_D) = -(\frac{\rho'_D \dot{H}}{3H} + \rho_D), \tag{8}$$

where $\rho' = \frac{d\rho_D}{dH}$, and dot denotes derivative with respect to time.

3. Universe evolution

In a flat FRW universe, Friedmann equations are

$$H^2 = \frac{8\pi}{3}(\rho_m + \rho_D), \tag{9}$$

$$H^2 + \frac{2}{3}\dot{H} = \frac{-8\pi}{3}(p_D),$$

where $\rho_m = \rho_0 a^{-3}$ and ρ_0 denote energy density of matter fields, and its value at the current era ($a = 1$), respectively. One can also obtain p_D as a function of H, by combining Eqs. (7) and (8) with the second Friedmann (9).

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