Initial conditions for critical Higgs inflation ☆

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A B S T R A C T

It has been pointed out that a large non-minimal coupling ξ between the Higgs and the Ricci scalar can source higher derivative operators, which may change the predictions of Higgs inflation. A variant, called critical Higgs inflation, employs the near-criticality of the top mass to introduce an inflection point in the potential and lower drastically the value of ξ. We here study whether critical Higgs inflation can occur even if the pre-inflationary initial conditions do not satisfy the slow-roll behavior (retaining translation and rotation symmetries). A positive answer is found: inflation turns out to be an attractor and therefore no fine-tuning of the initial conditions is necessary. A very large initial Higgs time-derivative (as compared to the potential energy density) is compensated by a moderate increase in the initial field value. These conclusions are reached by solving the exact Higgs equation without using the slow-roll approximation. This also allows us to consistently treat the inflection point, where the standard slow-roll approximation breaks down. Here we make use of an approach that is independent of the UV completion of gravity, by taking initial conditions that always involve sub-planckian energies.

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1. Introduction

So far no clear evidence for physics beyond the Standard Model at the scales explored by the LHC has been found. In this situation, it is useful to look for complementary tests. The extrapolation of the SM at energies much above those reachable at colliders offers a new way to look for further evidence of new physics (besides the already established ones, such as neutrino oscillations and dark matter).

Inflation is a natural arena to perform these tests. It was found that the Higgs of the SM might play the role of the inflaton provided that a sizable non-minimal coupling ξ with the Ricci scalar R is introduced. Ref. [1] considered originally the case of a very large ξ, which corresponds to the SM living well inside the so called stability region.[1] In this setup two different scales appear, the reduced Planck mass $\bar{M}_{\rm Pl} \simeq 2.435 \times 10^{18}$ GeV and $\bar{M}_{\rm Pl}/\xi$ and a violation of perturbative unitarity at $\bar{M}_{\rm Pl}/\xi$ has been found by considering scatterings of particles viewed as fluctuations around the EW vacuum [2,3]. This leads to the necessity of new physics or strong coupling methods to analyze that physical situation. While this does not undoubtedly exclude Higgs inflation (HI) as the rel-

evant expansion in that case is around a large Higgs field [4], the issue may be avoided by living very close to the boundary between stability and metastability,[2] the so called criticality. Indeed, a drastic decrease of ξ occurs at criticality [5–7], leading to a single new physics scale, $\bar{M}_{\rm Pl}$, where quantum gravity effects are expected to emerge.

Moreover, in [8] another issue of the large-ξ HI was pointed out. At the quantum level, it is necessary to tune the high energy values of some parameters in order to preserve the inflationary predictions: if this is not done large higher derivative terms in the effective action, such as R^2, are generated, changing the output of the model (see also [9]). However, Ref. [8] did not consider the critical HI case, which, as stated above, does not require a large ξ.

The aim of this article is to investigate whether critical HI suffers from any tuning in the choice of the high energy parameters. This will include in particular the analysis of the dependence of critical HI on the initial (pre-inflationary) conditions. Indeed, *any* slow-roll model of inflation, such as HI, should provide a mechanism that drives generic initial conditions to slow-rolling configurations, i.e. an inflationary attractor. If such an attractor does not exist a fine-tuning of the initial conditions is required, which makes the whole idea of inflation less attractive, given that its

[1] This is the region of parameter space where the electroweak (EW) vacuum is the global minimum of the SM effective potential.

[2] The metastability region is the region of parameter space where the lifetime of the EW vacuum exceeds the age of the universe.

main purpose is to solve fine-tuning problems (the horizon and flatness problems).

The paper is organized as follows. In Sec. 2 details of HI are given, including the classical analysis of inflation and a description of the quantum corrections; there, we will address the question of whether a fine-tuning of the high energy conditions of the running parameters is required in critical HI. In Sec. 3 we will consider initial conditions violating the slow-roll behavior in order to establish the existence of an inflationary attractor in HI; both analytical and numerical arguments will be used. Finally, Sec. 4 provides the conclusions.

2. The model

Let us define the Higgs inflation model [1]. The action is

$$S = \int d^4x \sqrt{-g} \left[\mathscr{L}_{\text{SM}} - \left(\frac{\bar{M}_{\text{Pl}}^2}{2} + \xi |H|^2 \right) R \right], \tag{1}$$

where H is the Higgs doublet, ξ is a real parameter and $\sqrt{-g}\mathscr{L}_{\text{SM}}$ is the SM Lagrangian minimally coupled to gravity. The part of the action that depends on the metric and the Higgs field *only* (the scalar-tensor part) is

$$S_{\text{st}} = \int d^4x \sqrt{-g} \left[|\partial H|^2 - V - \left(\frac{\bar{M}_{\text{Pl}}^2}{2} + \xi |H|^2 \right) R \right], \tag{2}$$

where $V = \lambda(|H|^2 - v^2/2)^2$ is the classical Higgs potential, and v is the EW Higgs vacuum expectation value. We assume a sizable non-minimal coupling, $\xi > 1$, because this is required by inflation as we will see.

2.1. Classical analysis

The $\xi |H|^2 R$ term can be eliminated through a *conformal* transformation (a.k.a. Weyl transformation):

$$g_{\mu\nu} \rightarrow \Omega^{-2} g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{2\xi |H|^2}{\bar{M}_{\text{Pl}}^2}. \tag{3}$$

The original frame, where the Lagrangian has the form in Eq. (1), is called the Jordan frame, while the one where gravity is canonically normalized (obtained with the transformation above) is called the Einstein frame. In the unitary gauge, where the only scalar field is the radial mode $\phi \equiv \sqrt{2|H|^2}$, we have (after having performed the conformal transformation)

$$S_{\text{st}} = \int d^4x \sqrt{-g} \left[K \frac{(\partial \phi)^2}{2} - \frac{V}{\Omega^4} - \frac{\bar{M}_{\text{Pl}}^2}{2} R \right], \tag{4}$$

and

$$K = \Omega^{-4} \left[\Omega^2 + \frac{3}{2} \left(\frac{d\Omega^2}{d\phi} \right)^2 \right]. \tag{5}$$

The non-canonical Higgs kinetic term can be made canonical through the field redefinition $\phi = \phi(\chi)$ defined by

$$\frac{d\chi}{d\phi} = \Omega^{-2} \sqrt{\Omega^2 + \frac{3}{2} \left(\frac{d\Omega^2}{d\phi} \right)^2}, \tag{6}$$

with the conventional condition $\phi(\chi = 0) = 0$. Note that $\phi(\chi)$ is invertible because Eq. (6) tells us $d\chi/d\phi > 0$. Thus, one can extract the function $\phi(\chi)$ by inverting the function $\chi(\phi)$ defined above.

Note that χ feels a potential

$$U \equiv \frac{V}{\Omega^4} = \frac{\lambda(\phi(\chi)^2 - v^2)^2}{4(1 + \xi \phi(\chi)^2/\bar{M}_{\text{Pl}}^2)^2}. \tag{7}$$

Let us now recall how slow-roll inflation emerges in this context. From (6) and (7) it follows [1] that U is exponentially flat when $\chi \gg \bar{M}_{\text{Pl}}$, which is a key property to have inflation. Indeed, for such high field values the quantities

$$\epsilon_U \equiv \frac{\bar{M}_{\text{Pl}}^2}{2} \left(\frac{1}{U} \frac{dU}{d\chi} \right)^2, \quad \eta_U \equiv \frac{\bar{M}_{\text{Pl}}^2}{U} \frac{d^2U}{d\chi^2} \tag{8}$$

are guaranteed to be small. Therefore, the region in field configurations where $\chi \gtrsim \bar{M}_{\text{Pl}}$ (or equivalently [1] $\phi \gtrsim \bar{M}_{\text{Pl}}/\sqrt{\xi}$) corresponds to inflation. In Sec. 3 we will investigate whether successful slow-roll inflation emerges also for large initial field kinetic energy. In this subsection we simply assume that the time derivatives are small. In this case, during the whole inflation the slow-roll parameters ϵ_U and η_U are small and the slow-roll approximation can be used.

All the parameters of the model can be determined with good accuracy through experiments and observations, including ξ [1,10]. ξ can be fixed by requiring that the measured curvature power spectrum (at horizon exit[3] $q = aH$) [11],[4]

$$P_R(q) \simeq (2.14 \pm 0.06) \times 10^{-9}, \tag{9}$$

is reproduced for a field value $\phi = \phi_{\text{b}}$ corresponding to an appropriate number of e-folds [10]:

$$N = \int\limits_{\phi_e}^{\phi_{\text{b}}} \frac{U}{\bar{M}_{\text{Pl}}^2} \left(\frac{dU}{d\phi} \right)^{-1} \left(\frac{d\chi}{d\phi} \right)^2 d\phi \simeq 59, \tag{10}$$

where ϕ_e is the field value at the end of inflation, computed by requiring

$$\epsilon(\phi_e) \simeq 1. \tag{11}$$

In the slow-roll approximation (used in this subsection) such constraint can be imposed by using the standard formula

$$P_R(k) = \frac{U/\epsilon_U}{24\pi^2 \bar{M}_{\text{Pl}}^4}. \tag{12}$$

For $N = 59$, this procedure leads to

$$\xi \simeq (5.02 \mp 0.06) \times 10^4 \sqrt{\lambda}, \qquad (N = 59) \tag{13}$$

where the uncertainty corresponds to the experimental uncertainty quoted in Eq. (9). Note that ξ depends on N:

$$\xi \simeq (4.61 \mp 0.06) \times 10^4 \sqrt{\lambda} \qquad (N = 54), \tag{14}$$

$$\xi \simeq (5.43 \mp 0.06) \times 10^4 \sqrt{\lambda} \qquad (N = 64). \tag{15}$$

Given that $\lambda \sim 0.1$, ξ has to be much larger than one at the classical level. The need of a very large ξ can be avoided when quantum corrections are included [5–7], as we will see in the next subsection.

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[3] We use a standard notation: a is the cosmological scale factor, $H \equiv \dot{a}/a$ and a dot denotes the derivative with respect to (cosmic) time, t.

[4] See for instance Table 3 of the second paper in Ref. [11] (P_R is denoted with A_s in that table). The value quoted here corresponds to the one with the smallest uncertainty in that table.