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Renormalized functional renormalization group

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ABSTRACT

We construct a new version of the effective average action together with its flow equation. The construction entails in particular the consistency of fluctuation field and background field equations of motion, even for finite renormalization group scales. Here we focus on the quantum gravity application, while the generalization of this idea to gauge theories is obvious. Our approach has immediate impact on the background field approximation, which is the most prominent approximation scheme within the asymptotic safety scenario. We outline the calculation of quantum gravity observables from first principles using the new effective average action.

1. Introduction

After more than 100 years of Einsteins theory of general relativity the search for the quantum theory of gravity is still one of the most important open problems in theoretical physics. Many different approaches are trying to shed light on this problem from various perspectives. Ultimately the fate of the various quantum gravity models is decided by experiments. Therefore, all these approaches have to make pre- or postdictions at some point. In this work we outline a practical method to calculate observables within the asymptotic safety scenario for quantum gravity [1,2].

To calculate observables of any theory of quantum gravity the expectation value $\langle \tilde{g} \rangle$ of the metric is of particular interest.[1] In order to derive $\langle \tilde{g} \rangle$ from first principles one has to solve the quantum equations of motion,

$$0 = \left. \frac{\delta \Gamma[g]}{\delta g} \right|_{g=\langle \tilde{g} \rangle}, \tag{1}$$

where Γ is the quantum effective action of gravity. Observables are then derived by combining the on-shell n-point correlators, $\Gamma^{(n)}[\langle \tilde{g} \rangle]$, into gauge invariant objects. Doing this for quantum gravity one can analyze the curvature invariants inside a black hole, to see what happens to the singularity, or one can study the time evolution of the metric to investigate cosmological inflation [3,4].

One of the key goals of asymptotic safety is the derivation of the effective average action Γ_k, which in the physical limit, $k \to 0$, approaches the quantum effective action, $\Gamma_k \xrightarrow{k \to 0} \Gamma$. This derivation involves two main steps: one first needs to find the eponymous asymptotically safe fixed point in the ultraviolet and in the second step one has to integrate the renormalization group flow down to the infrared leading to Γ. Under certain circumstances, e.g., for single scale problems, one can use Γ_k also for finite scales k instead of integrating down completely. The reason is, that in these cases the flow of the relevant correlators essentially stops shortly below the present physical scale.

In the literature the by far most used approximation scheme is the background field approximation, [2,5]. The main advantages are its seemingly manifest diffeomorphism invariance and a manageable amount of necessary calculations. So it was, e.g., possible to demonstrate that asymptotic safety actually is safe against the famous Goroff–Sagnotti counter term, which was believed to spoil the ultraviolet fixed point, as it marks the failure of perturbative quantum gravity, [6]. Despite the widespread use of the background field approximation, it still is an approximation. By now there are several works pointing towards quite some tension between proper fluctuation field calculations and the background field approximation, cf. [7], [8,9], [10–12] and [13]. These discrepancies are expected at least for nonzero renormalization group scales k, due to nontrivial split Ward identities, cf. equation (8) and [14]. Unfortunately, as discussed above, this is exactly what one would like to do: use Γ_k for finite scales k.

The reason why one actually has to track two separate fields is as follows. One can artificially split the full metric g into a back-

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[1] We use the tilde in $\langle \tilde{g} \rangle$ to indicate, that the $\tilde{g}$ is not a fixed metric, but the integration variable within a path integral, $\langle \tilde{g} \rangle \sim \int \mathcal{D}\tilde{g}\, \tilde{g}\, e^{-S_{\mathrm{cl}}[\tilde{g}]}$, where S_{cl} is the diffeomorphism invariant classical action of quantum gravity.

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ground metric $\bar{g}$ and a fluctuation field h by $g = \bar{g} + h$.[2] This simple split gets broken, due to the need for gauge fixing and regularization. Therefore naively Γ_k is a function of $\bar{g}$ and h separately, even though these fields are actually related by the nontrivial split Ward identities (8). This is nothing special to gravity, the same idea applies to gauge theories in the background field formulation, cf. [5,14,15]. However there, other than in gravity, one does not have to introduce a background.

In the literature there are essentially two kinds of approaches trying to investigate the separate dependence of Γ_k on $\bar{g}$ and h. The one kind deals with solving the split Ward identities in order to formulate the theory in terms of only one field, [16–24]. The other uses the fact, that if these split Ward identities are satisfied at a single renormalization group scale, then they are satisfied for all scales, if the flow is carried out in an exact manner. Therefore, one can in a first step forget about the identities and simply study the theory involving both fields, while at the end only making sure that the split Ward identities are satisfied in the infrared, [25–32]. Let us mention that there are also some geometric approaches, trying to deal with the separate dependence on the background and the fluctuation field in an explicitly gauge invariant manner [33–37].

In this letter we follow a new direction en route to the above problem. We present a simple modification of the standard effective average action in section 2. It guarantees that the equations of motion for the background field are compatible with the true quantum equations of motion of the full quantum field. Furthermore, this modification leads to several nice properties of the effective average action, cf. section 3. In particular we find a simple relation between the pure background effective average action and the remainder, which then also involves the fluctuation field. That is to say, a certain subset of the split Ward identities can be cast in a comparatively simple form. In this way we help to improve the understanding in both directions, solving the split Ward identities on the one hand and the study of both fields independently on the other hand.

The remainder of this section 1 can be safely skipped by readers not familiar with the functional renormalization group. We consider here the calculation of the expectation value of the metric, $\langle\tilde{g}\rangle_k$, to illustrate how the modification of the effective average action works. As $\langle\tilde{g}\rangle_k$ itself is not an observable, explicit calculations will depend on the choice of the background. However, it is to be expected that one gets a good estimate if one chooses the background such, that the expectation value of the fluctuation field vanishes [38]. This then corresponds to an expansion about the solution of the equations of motion. In this case the background field and the expectation value of the metric are identical. Hence, the defining equation for $\langle\tilde{g}\rangle_k$ is

$$0 = \left.\frac{\delta\Gamma_k[h;\bar{g}]}{\delta h}\right|_{\substack{h=0\\ \bar{g}=\langle\tilde{g}\rangle_k}} . \tag{2}$$

The new idea is to modify the effective average action Γ_k, such that the equations of motion for h and $\bar{g}$ are compatible at finite renormalization group scales k, without changing the quantum physics, i.e., the dynamics of h. We show in section 2, that this can be achieved by defining $\hat{\Gamma}_k$ as the Legendre transform,[3]

$$\hat{\Gamma}_k[h;\bar{g}] = \sup_J \left(J\cdot h - \hat{W}_k[J;\bar{g}]\right) - \Delta S_k[h;\bar{g}], \tag{3}$$

where $\hat{W}_k$ is a properly normalized Schwinger functional,

$$\hat{W}_k[J;\bar{g}] = \ln\frac{Z_k[J;\bar{g}]}{Z_k[0;\bar{g}]}, \tag{4}$$

with the partition function Z_k, cf. equation (10). The difference between Γ_k and $\hat{\Gamma}_k$ is the normalization $Z_k[0;\bar{g}]$ in the definition of the Schwinger functional (4). As $\hat{\Gamma}_k$ is built up entirely of elements already present in the standard formulation, the new flow equation is rather similar to the standard one, cf. equation (17).

It is important to note, that the normalization $Z_k[0;\bar{g}]$ in equation (4) only depends on the background. Therefore it does not have an impact on the fluctuation correlators containing the physics,

$$\frac{\delta\Gamma_k[h;\bar{g}]}{\delta h} = \frac{\delta\hat{\Gamma}_k[h;\bar{g}]}{\delta h}. \tag{5}$$

One can check that this additional background term ensures that the solution, $\langle\tilde{g}\rangle_k$, of equation (2) also is a solution of the analogous equation for the background field,

$$0 = \left.\frac{\delta\hat{\Gamma}_k[h;\bar{g}]}{\delta\bar{g}}\right|_{\substack{h=0\\ \bar{g}=\langle\tilde{g}\rangle_k}} . \tag{6}$$

Therefore, instead of calculating the expectation value of the metric using the fluctuation field, equation (2), we can equivalently use the background field, equation (6). Thus in future work one can improve the background field approximation by using $\hat{\Gamma}_k$.

2. Renormalized FRG

The asymptotic safety scenario relies on the idea of an interacting ultraviolet fixed point for quantum gravity. Therefore a perturbative treatment is no option. One way to investigate the properties of such an interacting fixed point is the study of the effective average action, Γ_k, together with the exact flow equation, cf. [2,39],

$$\dot{\Gamma}_k[h;\bar{g}] = \frac{1}{2}\,\mathrm{Tr}\left[\left(\Gamma_k^{(2;0)}[h;\bar{g}] + R_k[\bar{g}]\right)^{-1}\dot{R}_k[\bar{g}]\right]. \tag{7}$$

This equation is an explicit implementation of Wilson's renormalization group idea of integrating out the momenta shell-by-shell. Due to the specific properties of the regulator R_k the flow of Γ_k is driven by modes close to the renormalization group scale k. For $k\to 0$ the regulator vanishes and the effective average action approaches the full quantum effective action, $\Gamma_k \xrightarrow{k\to 0} \Gamma$.

By definition the information about quantum physics is contained in the correlators of the fluctuation field, while the background field is just a technical aid. As discussed earlier the simple linear split, $g = \bar{g} + h$, is broken due to the presence of the gauge fixing, S_{gf}, the ghosts, S_{gh}, and the regulator, $\Delta S_k[h;\bar{g}] = \frac{1}{2}h\cdot R_k[\bar{g}]\cdot h$, leading to the (modified) split Ward identities, cf. [14],

$$\left(\tfrac{\delta}{\delta h} - \tfrac{\delta}{\delta\bar{g}}\right)(\Gamma_k + \Delta S_k) = \left\langle\left(\tfrac{\delta}{\delta\hat{h}} - \tfrac{\delta}{\delta\bar{g}}\right)(S_{\mathrm{gf}} + S_{\mathrm{gh}} + \Delta S_k)\right\rangle_{\bar{g}}^{J}. \tag{8}$$

Here the $\langle\cdot\rangle_{\bar{g}}^{J}$ denotes the expectation value in presence of the sources, J, and the background field, $\bar{g}$. Due to the above identities the deviation of fluctuation and background field derivatives is only determined by unphysical terms, i.e., the regulator and the gauge fixing. Hence, the difference drops out when observables are calculated in the limit $k\to 0$. Therefore, once we have $\Gamma[h;\bar{g}]$, we can restrict ourselves to the gauge invariant functional $\Gamma[0;g]$ and use this to calculate all the observables. This insight lies at the

[2] We focus here on the linear split, while other splits can be discussed similarly.

[3] Here and in the following we mostly suppress the Faddeev–Popov ghosts as they are not important for our discussion.