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Scalar field configurations supported by charged compact reflecting stars in a curved spacetime

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ABSTRACT

We study the system of static scalar fields coupled to charged compact reflecting stars through both analytical and numerical methods. We enclose the star in a box and our solutions are related to cases without box boundaries when putting the box far away from the star. We provide bottom and upper bounds for the radius of the scalar hairy compact reflecting star. We obtain numerical scalar hairy star solutions satisfying boundary conditions and find that the radius of the hairy star in a box is continuous in a range, which is very different from cases without box boundaries where the radius is discrete in the range. We also examine effects of the star charge and mass on the largest radius.

1. Introduction

The no-scalar-hair theorem is a famous physical characteristic of black holes [1–3]. It was found that the static massive scalar fields cannot exist in asymptotically flat black holes, for references see [4–15] and a review see [16]. This property is usually attributed to the fact that the horizon of a classical black hole irreversibly absorbs matter and radiation fields. Along this line, one naturally want to know whether this no scalar hair behavior is a unique property of black holes. So it is interesting to explore possible similar no scalar hair theorem in other horizonless curved spacetimes.

Lately, hod found a no-scalar-hair theorem for asymptotically flat horizonless neutral compact reflecting stars with a single massive scalar field and specific types of the potential [17]. Bhattacharjee and Sudipta further extended the discussion to spacetimes with a positive cosmological constant [18]. In fact, the no scalar hair behavior also exists for massless scalar field nonminimally coupled to gravity on the neutral compact reflecting star background [19]. Recently, scalar field configurations were constructed in the charged compact reflecting shell where the star charge and mass can be neglected compared to the star radius [20,21]. With analytical methods, the physical properties of the asymptotically flat composed star-field configurations were also analyzed in [22]. In particular, this work derived a remarkably compact analytical formula for the discrete spectrum of star radii. Along this line, it is interesting to extend the discussion by relaxing the condition that star radii are much larger than the star charge and mass.

On the other side, a simple way to invade the black hole no-scalar-hair theorem is adding a reflecting box boundary. It should be emphasized that the boundary conditions imposed by a box are different from the familiar boundary conditions of asymptotically flat spacetimes. In fact, it was found that the low frequency scalar field perturbation can trigger superradiant instability of the charged black hole in a box and the nonlinear dynamical evolution can form a quasi-local hairy black hole [23–26]. From thermodynamical aspects, Pallab and other authors showed that there are stable asymptotically flat hairy black holes in a box invading no-hair-theorem of black holes [27]. It was believed that the box boundary could play a role of the infinity potential to make the fields bounce back and condense around the black hole. Along this line, it is interesting to extend the discussion of scalar field configurations supported by a compact reflecting star through including an additional box boundary and also compare mathematical structures between gravities without box boundaries and models in a box.

The next sections are planed as follows. In section 2, we introduce the model of a charged compact reflecting star coupled to a scalar field. In section 3.1, we provide bounds for the radius of the scalar hairy star. In section 3.2, we obtain radius of the hairy star and explore effects of parameters on the largest radius. And in section 3.3, we also carry out an analytical study of the system in the limit that star charge and mass can be neglected. We will summarize our main results in the last section.

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2. Equations of motion and boundary conditions

We consider the system of a scalar field and a compact reflecting star enclosed in a time-like reflecting box at $r = r_b$ in the four dimensional asymptotically flat gravity. When $r_b \to \infty$, we go back to the case without box boundaries. We also define the radial coordinate $r = r_s$ as the radius of the compact star. And the corresponding Lagrange density is given by

$$\mathcal{L} = -\frac{1}{4}F^{MN}F_{MN} - |\nabla_\mu \psi - qA_\mu \psi|^2 - \mu^2\psi^2, \tag{1}$$

where q and μ are the charge and mass of the scalar field $\psi(r)$ respectively. And A_μ stands for the ordinary Maxwell field.

Using the Schwarzschild coordinates, the line element of the spherically symmetric star can be expressed in the form [28]

$$ds^2 = -(1 - \frac{2M}{r} + \frac{Q^2}{r^2})dt^2 + \frac{dr^2}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}} + r^2(d\theta^2 + sin^2\theta d\varphi^2), \tag{2}$$

where M is the mass of the star and Q is the charge of the star. In this paper, we only study the case of $M \geqslant Q$. Since the spacetime is regular, we also assume that $r_s > M + \sqrt{M^2 - Q^2}$. And the Maxwell field with only the nonzero tt component is $A_\mu = -\frac{Q}{r}dt$.

For simplicity, we study the scalar field with only radial dependence in the form $\psi = \psi(r)$. From above assumptions, we obtain equations of motion as

$$\psi'' + (\frac{2}{r} + \frac{g'}{g})\psi' + (\frac{q^2Q^2}{r^2g^2} - \frac{\mu^2}{g})\psi = 0, \tag{3}$$

with $g = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$.

In addition, we impose reflecting boundary conditions for the scalar field at the surface of the compact star. We also suppose that the time-like box boundary $r = r_b$ can reflect the scalar field back. So the scalar field vanishes at the boundaries as

$$\psi(r_s) = 0, \qquad \psi(r_b) = 0. \tag{4}$$

3. Scalar field configurations in charged compact reflecting stars

3.1. Bounds for the radius of the scalar hairy compact star

Defining the new radial function $\tilde{\psi} = \sqrt{r}\psi$, one obtains the differential equation

$$r^2\tilde{\psi}'' + (r + \frac{r^2g'}{g})\tilde{\psi}' + (-\frac{1}{4} - \frac{rg'}{2g} + \frac{q^2Q^2}{g^2} - \frac{\mu^2r^2}{g})\tilde{\psi} = 0, \tag{5}$$

with $g = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$.

According to the boundary conditions (4), one deduce that

$$\tilde{\psi}(r_s) = 0, \qquad \tilde{\psi}(r_b) = 0. \tag{6}$$

The function $\tilde{\psi}$ must have (at least) one extremum point $r = r_{peak}$ between the surface r_s of the reflecting star and the box boundary r_b (including cases of $r_b = \infty$). At this extremum point, the scalar field is characterized by

$$\{\tilde{\psi}' = 0 \quad \text{and} \quad \tilde{\psi}\tilde{\psi}'' \leqslant 0\} \quad \text{for } r = r_{peak}. \tag{7}$$

According to the relations (5) and (7), we arrive at the inequality

$$-\frac{1}{4} - \frac{rg'}{2g} + \frac{q^2Q^2}{g^2} - \frac{\mu^2r^2}{g} \geqslant 0 \quad \text{for } r = r_{peak}. \tag{8}$$

Then we have

$$\mu^2r^2g \leqslant q^2Q^2 - \frac{rgg'}{2} - \frac{1}{4}g^2 \quad \text{for } r = r_{peak}. \tag{9}$$

Since $r \geqslant r_s > M + \sqrt{M^2 - Q^2} \geqslant M \geqslant Q$, we have

$$g = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{1}{r^2}(r^2 - 2Mr + Q^2) = \frac{1}{r^2}[(r - M)^2 - (M^2 - Q^2)] \geqslant 0, \tag{10}$$

$$rg' = r(1 - \frac{2M}{r} + \frac{Q^2}{r^2})' = r(\frac{2M}{r^2} - \frac{2Q^2}{r^3}) = \frac{2M}{r}(1 - \frac{Q}{r}\frac{Q}{M}) \geqslant 0 \tag{11}$$

and

$$(r^2g)' = (r^2 - 2Mr + Q^2)' = 2(r - M) \geqslant 0. \tag{12}$$

Then we arrive at

$$\mu^2r_s^2g(r_s) \leqslant \mu^2r^2g(r) \leqslant q^2Q^2 - \frac{rgg'}{2} - \frac{1}{4}g^2 \leqslant q^2Q^2 \quad \text{for } r = r_{peak}. \tag{13}$$

According to (13), there is

$$\mu^2r_s^2g(r_s) \leqslant q^2Q^2. \tag{14}$$

Taking cognizance of the metric solutions, (14) can also be expressed as

$$\mu^2r_s^2(1 - \frac{2M}{r_s} + \frac{Q^2}{r_s^2}) \leqslant q^2Q^2. \tag{15}$$

Then, we can transfer (15) into the form

$$(\mu r_s)^2 - (2\mu M)(\mu r_s) + Q^2(\mu^2 - q^2) \leqslant 0. \tag{16}$$

Then, we obtain bounds for the radius of the scalar hairy compact reflecting star as

$$\mu M + \sqrt{\mu^2(M^2 - Q^2)} < \mu r_s \leqslant \mu M + \sqrt{\mu^2(M^2 - Q^2) + q^2Q^2}. \tag{17}$$

The bottom bound comes from the assumption that the spacetime is regular or $r_s > M + \sqrt{M^2 - Q^2}$ and the upper bound can be obtained from (16). For a neutral scalar field with $q = 0$, (17) shows that the upper bound is behind a horizon meaning a no-hair-theorem for the neutral scalar field in a charged reflecting star. So it is the coupling qQ makes the upper bound larger than the horizon critical points and then the scalar hair can possibly exist in this regular spacetime.

3.2. Scalar field configurations in a curved spacetime

The scalar field configurations with charged reflecting stars were studied in the limit of $Q, M \ll r_s$ [20–22]. In this part, we will extend the discussion by relaxing the condition $Q, M \ll r_s$. We can simply set $\mu = 1$ in the following calculation using the symmetry of the equation (3) in the form

$$r \to kr, \quad \mu \to \mu/k, \quad M \to kM, \quad Q \to kQ, \quad q \to q/k. \tag{18}$$

Around the star surface, the scalar field can be expanded as $\psi = \psi_0(r - r_s) + \cdots$. Since the scalar field equation is linear and homogeneous with respect to ψ, we can fix $\psi_0 = 1$ and use the