



Perturbative Yang–Mills theory without Faddeev–Popov ghost fields

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ARTICLE INFO

Article history:

Received 8 February 2018

Received in revised form 23 February 2018

Accepted 13 March 2018

Available online 14 March 2018

Editor: N. Lambert

ABSTRACT

A modified Faddeev–Popov path integral density for the quantization of Yang–Mills theory in the Feynman gauge is discussed, where contributions of the Faddeev–Popov ghost fields are replaced by multi-point gauge field interactions. An explicit calculation to $O(g^2)$ shows the equivalence of the usual Faddeev–Popov scheme and its modified version.

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1. Introduction

Faddeev and Popov [1] proposed a highly acclaimed path integral quantization procedure for Yang–Mills theory. Yang–Mills theory is a gauge theory based on compact simple Lie groups. It forms the basis of our understanding of the Standard Model of particle physics [2–4], which has two basic components: The spontaneously broken $SU(2) \times U(1)$ electroweak theory, and the unbroken $SU(3)$ color gauge theory, known as Quantum Chromodynamics (QCD).

Although electromagnetism and the weak interactions (responsible for the forces between sub-atomic particles that cause radioactive decay) appear quite different at everyday low energies, the Standard Model understands them as two different aspects of the same force. The Higgs mechanism [5–9] provides an explanation for the presence of massive gauge bosons (the carriers of the weak force) aside of the massless photon (the carrier of the electromagnetic force). The discoveries of the massive W^\pm and Z gauge bosons at the CERN $\bar{p}p$ collider [10–13] as well as of the Higgs particle at the Large Hadron Collider [14,15] are considered as major successes for the European Organization for Nuclear Research.

QCD is the theory which describes the strong interactions between massive quarks and massless gluons (responsible for binding neutrons and protons to create atomic nuclei). QCD exhibits two main properties, asymptotic freedom [16,17] and color confinement. Asymptotic freedom refers to the weakness of the strong interactions at short-distances (or high energies, respectively). It allows a perturbative treatment, which is often referred to as perturbative QCD. Hereby high energy hadronic processes involving a

large momentum transfer can be factorized into one part which requires detailed nonperturbative information on parton distribution functions and into a second part, which is calculable using perturbation theory. Parton distribution functions (specifying how hadrons are built out of quarks and gluons) have to be extracted from data and are available from various groups worldwide. The perturbative part of the calculation is done in an expansion in the coupling constant. In this context, we especially mention results at next-to-leading order [18], next-to-next-to-leading order [19–22], even next-to-next-to-next-to-leading order [23,24], as well as calculations supplemented with resummations of logarithmic contributions [25,26]. Confinement is the phenomenon of non-observation of color charged particles like free quarks or gluons and is believed to follow from the strength of the QCD coupling constant at long distances (or low energies, respectively). It should be remarked, however, that presently there is no analytic proof of color confinement in Yang–Mills theory. Confinement is crucial for explaining why nuclear forces are short ranged while massless gluon exchange would be long ranged: Nucleons are colorless so they cannot exchange colored gluons but only colorless states. The lightest such particles are pions, which fixes the range of nuclear forces by the inverse of their mass to about 10^{-14} cm.

Upon quantizing Yang–Mills theory new fields are introduced, called Faddeev–Popov ghost fields, which are associated to the gauge fixing. Mathematically, ghost fields allow for an integral representation of the Faddeev–Popov determinant (see below) in terms of a local action functional. The ghost fields are Lorentz scalars but obey Fermi statistics, they are arising inside Feynman diagrams in closed loops only. The ghosts' unphysical degrees of freedom are needed to exactly cancel unphysical polarization states of the gauge field, leading to a unitary theory. The proof of unitarity relies on the Slavnov–Taylor identities [27,28], which in turn play a key role in the proof of the renormalizability [29,30]

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of Yang–Mills theories. The Slavnov–Taylor identities led Becchi, Rouet and Stora [31,32] and, independently, Tyutin [33] to discover a global supersymmetry invariance of the gauge fixed Yang–Mills action including the ghost contributions.

The perturbation theory of Yang–Mills theory as developed by Faddeev–Popov, the property of asymptotic freedom and renormalizability are at the heart of the Standard Model of elementary particle physics.

In this paper a modified Faddeev–Popov path integral quantization of Yang–Mills theory is presented, where contributions of the Faddeev–Popov ghost fields are replaced by multi-point gauge field interactions. This is a new formulation of quantum Yang–Mills theory without the use of Grassman-valued fields.

2. Local features of Yang–Mills theory

Let \mathcal{A} be the space of Yang–Mills fields and \mathcal{G} the gauge group (for a detailed mathematical account of the involved space we refer to [34–36]). Then \mathcal{G} defines a principal \mathcal{G} -bundle $\mathcal{A} \xrightarrow{\pi} \mathcal{A}/\mathcal{G} =: \mathcal{M}$ over the space \mathcal{M} of all inequivalent gauge potentials with projection π . \mathcal{M} represents the true degrees of freedom. However, the principal \mathcal{G} -bundle $\mathcal{A} \rightarrow \mathcal{M}$ is not globally trivializable [34–36], giving rise to the so-called Gribov ambiguity [37] (for a recent review see [38]).

It is advantageous to separate the Yang–Mills fields A_μ into gauge independent and gauge dependent degrees of freedom. As this is only locally possible due to the non triviality of the bundle $\mathcal{A} \rightarrow \mathcal{M}$, we consider the trivializable bundle $\pi^{-1}(U) \rightarrow U$, where U denotes a sufficiently small neighborhood in \mathcal{M} .

Under a gauge transformation $\Omega \in \mathcal{G}$ the Yang–Mills field transforms according to

$$A_\mu^\Omega = \Omega A_\mu \Omega^{-1} - \frac{i}{g} (\partial_\mu \Omega) \Omega^{-1}, \tag{1}$$

where g denotes the Yang–Mills coupling constant. In terms of the local gauge fixing surface

$$\Gamma = \{B_\mu \in \pi^{-1}(U) \mid \partial_\mu B_\mu = 0\} \tag{2}$$

all gauge fields in $\pi^{-1}(U)$ have the form B_μ^Ω , where $B_\mu \in \Gamma$ and $\Omega \in \mathcal{G}$. Conversely, given any $A_\mu \in \pi^{-1}(U)$, there exists a uniquely defined $\Omega(A) \in \mathcal{G}$ such that $A_\mu^{\Omega(A)^{-1}} \in \Gamma$. This explicitly means that $\Omega(A)$ has to obey

$$0 = \partial_\mu (\Omega(A)^{-1} A_\mu \Omega(A)) - \frac{i}{g} \partial_\mu ((\partial_\mu \Omega(A)^{-1}) \Omega(A)). \tag{3}$$

This equation may be solved for $\Omega(A)$ as a formal power series in the gauge field A_μ [39–42], which will be used in the next section.

3. Perturbative Yang–Mills theory without Faddeev–Popov ghost fields

To begin with, let us recall the usual Faddeev–Popov formula [1] for calculating expectation values of gauge invariant observables f

$$\langle f \rangle = \frac{\int \mathcal{D}A \delta(\partial_\mu A_\mu) \det \mathcal{F}_A e^{-S_{\text{inv}}[A]} f(A)}{\int \mathcal{D}A \delta(\partial_\mu A_\mu) \det \mathcal{F}_A e^{-S_{\text{inv}}[A]}}, \tag{4}$$

displaying integrations over unconstrained gauge fields $A_\mu \in \pi^{-1}(U)$ and delta functions imposing the gauge fixing condition. Here $\det \mathcal{F}_A = \det \partial_\mu D_\mu(A)$ denotes the determinant of the Faddeev–Popov operator and $D_\mu(A)$ is the covariant derivative with respect to A_μ . The gauge invariant Yang–Mills action $S_{\text{inv}}[A]$ reads

$$S_{\text{inv}}[A] = \frac{1}{2} \int d^d x \text{Tr} (F_{\mu\nu} F_{\mu\nu}) \tag{5}$$

and is defined in terms of the field strength

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]. \tag{6}$$

Note that in order to arrive at the Faddeev–Popov formula [1] an infinite gauge group volume had to be canceled between the numerator and denominator of the expression (4).

We prefer to represent the Faddeev–Popov formula (4) in the equivalent form [43,44]

$$\langle f \rangle = \frac{\int \mathcal{D}B \det \mathcal{F}_B e^{-S_{\text{inv}}[B]} f(B)}{\int \mathcal{D}B \det \mathcal{F}_B e^{-S_{\text{inv}}[B]}}, \tag{7}$$

where the path integral is performed over constrained gauge fields $B_\mu \in \Gamma$, and where $\det \mathcal{F}_B = \det \partial_\mu D_\mu(B)$ denotes the determinant of the Faddeev–Popov operator with respect to B_μ .

Inspired by the stochastic quantization scheme [45–47] a generalization of the Faddeev–Popov formula was proposed in [48], where

$$\langle f \rangle = \frac{\int \mathcal{D}B \det \mathcal{F}_B e^{-S_{\text{inv}}[B]} f(B) \int \mathcal{D}\Omega e^{-S_G[\Omega]}}{\int \mathcal{D}B \det \mathcal{F}_B e^{-S_{\text{inv}}[B]} \int \mathcal{D}\Omega e^{-S_G[\Omega]}}. \tag{8}$$

Here $S_G \in C^\infty(\mathcal{G})$ is an arbitrary functional on \mathcal{G} , such that e^{-S_G} is integrable with respect to the invariant measure $\mathcal{D}\Omega$ on \mathcal{G} . For different modifications of the Faddeev–Popov formula see [40,49].

When evaluated on gauge invariant observables all additional finite contributions of the gauge degrees of freedom due to S_G cancel out, therefore the generalized definition (8) of expectation values equals (7), which in turn is equivalent to the usual Faddeev–Popov formula (4).

It is our intention, however, *not* to cancel these finite contributions, but to transform the fields $B_\mu \in \Gamma$ and $\Omega \in \mathcal{G}$ back into the original variables $A_\mu \in \pi^{-1}(U)$. In this case the Jacobian of the field transformation eliminates the Faddeev–Popov determinant, so we obtain [48]

$$\langle f \rangle = \frac{\int \mathcal{D}A e^{-S_{\text{inv}}[A] - S_G[\Omega(A)]} f(A)}{\int \mathcal{D}A e^{-S_{\text{inv}}[A] - S_G[\Omega(A)]}}. \tag{9}$$

Mind that now the path integral is performed over unconstrained gauge fields $A_\mu \in \pi^{-1}(U)$, similarly as in (4). Due to the absence of the Faddeev–Popov determinant, however, Faddeev–Popov ghost fields are not present any longer.

In this work we suggest to specify S_G as

$$S_G[\Omega(A)] = \frac{1}{g^2} \int d^d x \text{Tr} \left((\partial_\mu \theta(A)_\mu) (\partial_\nu \theta(A)_\nu)^\dagger \right), \tag{10}$$

where

$$\theta(A)_\mu = (\partial_\mu \Omega(A)^{-1}) \Omega(A) \tag{11}$$

is defined in terms of $\Omega(A)$.

To accommodate in $S_G[\Omega(A)]$ the explicit expression for $\Omega(A)$ is an involved task and can only be achieved in a perturbative expansion in the coupling constant g . Although $S_G[\Omega(A)]$ is depending on the gauge fields A_μ in a highly intricate manner, the path integral (9) itself is performed over unconstrained gauge fields $A_\mu \in \pi^{-1}(U)$.

It will be seen that our choice for S_G is implying gauge field propagators in the Feynman gauge. In a sequel paper we plan to study also the covariant ξ -gauges as well as the limiting $\xi \rightarrow 0$ case of the Landau gauge, when multiplying S_G by the inverse of a gauge fixing parameter ξ .

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