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Topological transport from a black hole

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A B S T R A C T

In this paper the low temperature zero-frequency transport in a $2+1$-dimensional theory dual to a dyonic black hole is discussed. It is shown that transport exhibits topological features: the transverse electric and heat conductivities satisfy the Wiedemann–Franz law of free electrons; the direct heat conductivity is measured in units of the central charge of CFT_{2+1}, while the direct electric conductivity vanishes; the thermoelectric conductivity is non-zero at vanishing temperature, while the $O(T)$ behavior, controlled by the Mott relation, is subleading. Provided that the entropy of the black hole, and the dual system, is non-vanishing at $T=0$, the observations indicate that the dyonic black hole describes a $\hbar \to 0$ limit of a highly degenerate topological state, in which the black hole charge measures the density of excited non-abelian quasiparticles. The holographic description gives further evidence that non-abelian nature of quasiparticles can be determined by the low temperature behavior of the thermoelectric transport.

Introduction. AdS/CFT is a powerful tool to approach a certain class of strongly coupled quantum systems. The method is based on a conjectured duality between string theory in anti-de Sitter (AdS) space and conformal theory (CFT) on the boundary of AdS [1]. When the string theory is in its low-energy weak-coupling limit of classical gravity the dual CFT is in a quantum strongly coupled phase. Henceforth we refer to this as the *holographic* limit.

It turns out that the strong coupling regime of the CFT probed by the duality is a peculiar one. In particular, it does not apply directly to strong interactions in particle physics, as originally expected, since it addresses the regime of extreme number of internal degrees of freedom (color) and extreme values of coupling constant. It was consequently proposed that a more natural domain of applicability of AdS/CFT belongs to condensed matter physics. An appropriate reviews can be found in Refs. [2,3].

Following the idea of the proposal we would like to revisit the view of AdS/CFT on transport in $2+1$-dimensional systems with finite charge density and magnetic field. The focus in this paper will be on the low-temperature transport. Based on the analysis of transport properties we claim that the simplest $3+1$-dimensional dual gravity description of such a system predominantly reflects its topological features.

The most interesting observation that we will present here is that $3+1$D/$2+1$D-dimensional "holographic" duality is consistent with the $2+1$D/$1+1$D bulk-to-boundary correspondence in well-known topological setups, such as quantum Hall effect (QHE). Most transparently, the heat conductivities, as computed by the gravity model, exhibit a typical behavior, consistent with CFT models of $1+1$-dimensional edge modes in QHE. The low temperature results thus indicate exact systems, where predictions of AdS/CFT could be tested, even experimentally. Some experimental challenges are outlined in the conclusion to this paper. In particular, holography instructs us to work in a "classical" regime of degenerate topological states of matter.

Although the analysis in this letter is based on the results known in the literature, low-temperature consequences that we present here have not been previously discussed, to the best of our knowledge. We believe that our findings, such as exact Lorenz ratio, Mott relation for the Seebeck coefficient and its low-temperature value are new and might be of interest for condensed matter community. Furthermore we suggest a topological interpretation of the extremal black hole entropy, which may resolve a long-standing issue with the degeneracy of the holographic ground state at zero temperature.

The system that has the above features is provided by an electrically and magnetically charged (dyonic) black hole [4].

In dyonic black holes transport was originally discussed by Hartnoll and Kovtun [4], and later by seminal papers [5] and [6] of Hartnoll et al., which demonstrated an impressive consis-

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tence of holographic approach with a more conventional hydrodynamical one. The gravity side of the story is provided by a $3+1$-dimensional Einstein–Maxwell theory with a negative cosmological constant. This theory has a solution corresponding to an asymptotically AdS black hole metric coupled to electric and magnetic fields. The latter fields are parallel to the fourth "radial" AdS coordinate z, so that at the asymptotic boundary $z \to 0$, where the expected $2+1$-dimensional dual theory lives, the electric field turns into a two-dimensional surface charge density ρ, while the magnetic field B becomes a transverse magnetic flux.

Following the holographic prescription one can compute equilibrium thermodynamics of the dual system as well as response to external perturbations. Gravity calculation appears as powerful as the hydrodynamical one, yet its results extend beyond the hydrodynamic approximation. Summarizing the zero frequency results from Refs. [4,5] on transport coefficients, classical gravity calculation in the dyonic black hole background expresses electric, thermal and mixed conductivities in terms of the thermodynamical quantities. While the result for electric conductivity does not seem to be very illuminating, namely, non-vanishing is only the transverse Hall conductivity, which is expressed as $\sigma_H = \rho/B$, the thermal conductivity is given by a less trivial expression:

$$\kappa_{xx} = \kappa_{yy} = \frac{as^2T}{\rho^2 + a^2B^2}\,,$$
$$\kappa_{xy} = -\kappa_{yx} = \frac{\rho s^2 T}{B(\rho^2+a^2B^2)}\,. \tag{1}$$

Here T and s are the temperature and entropy density of the thermodynamic system described by the black hole. The quantity

$$a = \frac{L^2}{4G}\,, \tag{2}$$

comes from the gravitational/geometric parameters: G is the four-dimensional Newton's constant and L is the curvature radius of the AdS space. One can establish the precise meaning of a on the dual side, if the black hole is embedded in a full string theory setup. In [7] it is identified as $\sqrt{2}N^{3/2}/6\pi$ in terms of a dual superconformal gauge theory with $SU(N)$ gauge group. More generally it is a parameter that characterizes a number of degrees of freedom of the dual CFT.

In the low temperature limit thermodynamics of the dyonic black hole and Eq. (1) yield the following result for the thermal conductivities

$$\kappa_{xx} = \frac{\pi^2}{3}aT + O(T^2)\,,$$
$$\kappa_{xy} = \frac{\pi^2}{3}\sigma_H T + O(T^2)\,. \tag{3}$$

The numerical coefficient $\pi^2/3$ that appears in the expressions for the conductivities is the conventional quantum of thermal conductivity "quantized" in units of parameters a and σ_H. In particular, it was appreciated in Ref. [8] that the ratio of transverse thermal and electric conductivities,

$$\frac{\kappa_{xy}}{\sigma_{xy}} = \frac{\pi^2}{3}T\,, \tag{4}$$

yields the Wiedemann–Franz law for classical metals.

The expression for a in terms of gravity parameters together with the form it appears in Eq. (3) implies that we should identify a with a central charge of the dual theory. In three-dimensional gravity in AdS space their exists a similar relation derived by Brown and Henneaux [9]. Specifically, the boundary degrees of freedom of AdS_3 are governed by a $1+1$-dimensional CFT with central charge $c = 3L^{(3D)}/2G^{(3D)}$. Expression (2) for a is a particular $D = 2+1$ form of the central charge in odd dimensions conjectured by Myers and Sinha [10] in the analysis of the universal contribution to entanglement entropy. Eq. (3) provides further evidence to this conjecture.

In a topological state, such as one in QHE, transport occurs at edges of the system. In the presence of edges the effective Chern–Simons theory of QHE requires massless boundary degrees of freedom to preserve gauge invariance. The theory of edge modes is a CFT, whose central charge is connected with the filling fraction of the QHE state [11]. CFT allows to compute the transverse Leduc–Righi (LR) conductivity

$$\kappa_{xy} = \frac{\pi^2}{3}\nu_Q T\,, \tag{5}$$

where ν_Q is a sum over edge mode channels. In integer QHE, see *e.g.* [12], $\nu_Q = \sigma_H$, and the holographic formula for the LR conductivity appears consistent. The difference between ν_Q and σ_H appears when channels with different quasiparticle charges and, consequently, different σ_H are present. This is not captured by the naive holographic picture.

In a 1998 paper [13] Green and Read proposed to derive the quantization of κ_{xy}, coupling energy current to external metric fluctuations. Similarly to the electric potential, metric would be controlled by a gravitational Chern–Simons theory. As we know, gravity has a Chern–Simons description in three dimensions. The approach suggested by Green and Read is what is now "routinely" applied in AdS/CFT.

In 3D the above result for the LR conductivity is easy to obtain. In AdS_3 an easy calculation yields

$$\kappa = \frac{\pi}{6}cT\,, \tag{6}$$

in terms of the Brown–Henneaux central charge c. Note that $2+1$-dimensional gravity describes a $1+1$-dimensional system, which is the edge of the QHE bar. Thus $\kappa = \kappa_{xy}$ and $\sigma_H = c/2\pi$. Meanwhile the direct conductivity κ_{xx} is defined in terms of a $2+1$-dimensional central charge a.

The agreement between Eqs. (3) and (6) is quite interesting. We remind that while in AdS_3 this result can be easily obtained using conformal symmetry, in AdS_4 it becomes a much less trivial calculation, *e.g.* [4]. We believe that the reason for the agreement lies in the fact that the $T \to 0$ result is topological.

In experiment more accessible are the thermoelectric coefficients, which characterize the current or voltage response to an applied temperature difference. First, from formulae in [4,5] one finds the low temperature expansion of the thermoelectric conductivity:

$$\alpha_{xx} = 0\,,$$
$$\alpha_{xy} = -\alpha_{yx} = \frac{\pi}{\sqrt{3}}\sqrt{\sigma_H^2 + a^2} + O(T)\,. \tag{7}$$

It appears that the off-diagonal part of α is a square root of the sum of κ_{xx}^2 and κ_{xy}^2 divided by the temperature.

The thermoelectric power (TEP) S can be found from the matrix formula $S = -\sigma^{-1}\cdot\alpha$. We conclude that

$$S_{xx} = -\frac{s}{\rho} = -\frac{\pi}{\sqrt{3}}\frac{\sqrt{\sigma_H^2 + a^2}}{\sigma_H} + O(T)\,, \tag{8}$$