Annals of Physics 395 (2018) 219–237

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Qualitative analysis of magnetic waveguides for two-dimensional Dirac fermions

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ARTICLE INFO

Article history:
Received 15 February 2018
Accepted 29 May 2018
Available online 1 June 2018

Keywords:
Dirac material
Magnetic waveguide
Qualitative analysis

ABSTRACT

We focus on the confinement of two-dimensional Dirac fermions within the waveguides created by realistic magnetic fields. Understanding of their band structure is of our main concern. We provide easily applicable criteria, mostly depending only on the asymptotic behavior of the magnetic field, that can guarantee existence or absence of the energy bands and provide valuable insight into the systems where analytical solution is impossible. The general results are employed in specific systems where the waveguide is created by the magnetic field of a set of electric wires or magnetized strips.

1. Introduction

A great variety of condensed matter systems can host two-dimensional Dirac fermions. The list of the so called Dirac materials [1] contains not only graphene, but also other systems where relativistic quasi-particles were either predicted or even confirmed experimentally. Let us mention silicene, germanene, dichalcogenides [2–5], or artificial crystal lattices synthesized in the lab with the use of optical traps [6–8] or molecular manipulations [9,10]. Due to their remarkable characteristics, Dirac materials (and graphene in particular) are hoped to be the building blocks of the post-silicon electronics. This sets a great motivation to understand and control their electronic properties.

Massless Dirac fermions can propagate through electrostatic barriers without being back-scattered. This phenomenon, Klein tunneling, challenges effective control of the quantum transport with the use of the electric field. Therefore, an alternative approach is needed. Cutting the sample into the required form or chemical adsorptions of other atoms (e.g. oxygen, fluoride) is demanding on precision and

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https://doi.org/10.1016/j.aop.2018.05.021

lacks flexibility. In this respect, control of the Dirac fermions by magnetic or magneto-electric barriers represents a more feasible option [11].

Dynamics of two-dimensional (non-relativistic) electron gas (2DEG) in an inhomogeneous magnetic field was studied already in [12–14], three-dimensional Dirac fermions in [15]. It was found that when otherwise homogeneous magnetic field changes its sign along a straight line, the electrons can propagate along the line with a non-vanishing group velocity. This is in coherence with the classical picture, where electrons propagate along the trajectories that are bending back and forth along the interface. These states are called snake states in the literature.

Confinement of Dirac fermions in graphene by magnetic fields was proposed in [16,17]. Propagation both across and along the magnetic barriers was studied in great number of papers so that we have no hope to provide complete list of the related references. The wave vector filtering known for 2DEG [14], where the electrons bouncing on the magnetic barrier are totally reflected for a broad interval of angles, was analyzed for Dirac fermions, e.g., in [18–22]. Existence of snake states in graphene was addressed in [23,24]. Effect of many-body interactions was discussed in [25], spin snake states were discussed in [26]. The existence of snake states in graphene was also studied both theoretically and experimentally in the systems where both magnetic and electric fields change abruptly along a straight line [27–31], see also the review article [11].

The systems studied in the literature usually possess translational invariance in one direction. It implies conservation of the (longitudinal) momentum k along the barrier. The energy spectrum of the two-dimensional system can be reconstructed from the spectra of effectively one-dimensional subsystems with fixed k. In many of the mentioned works, the external fields are represented by rectangular barriers or step functions [13–16,18–20,29,25,28,24]. The spectra of their one-dimensional subsystems contain eigenvalues at least for some values of k. Then there are energy bands $E_n(k)$ in the spectrum of the full two-dimensional Hamiltonian. The energy bands can coincide with Landau levels for large $|k|$ (the case when the magnetic field is asymptotically constant and of the same sign at infinities) or there can also emerge dispersive states localized at the barrier and moving along it with a non-vanishing group velocity $v_n = \partial_k E_n(k)$, [13,15]. The solvable models with step-like vector potential were discussed e.g. in [21,27]. The effectively one-dimensional subsystem has non-vanishing essential spectrum and possibly also the discrete eigenvalues that correspond to the states confined at the barrier. The essential spectra build up the typical wedges of allowed energies in the spectrum of the full Hamiltonian, while the discrete energy levels of the subsystem form the energy bands $E_n(k)$ [27].

The energy bands $E_n(k)$ can be associated with the existence of wave packets that do not disperse in transverse direction [32]. When the group velocity $v_n(k)$ acquires both positive and negative values, the barrier represents a waveguide for *bidirectional* quantum transport of the wave packets. When $v_n(k)$ is strictly positive or negative, then a *unidirectional* quantum transport of the dispersionless wave packets along the barrier takes place [27].

The analysis of solvable models was not restricted to the potentials in the form of step-like functions. The techniques of supersymmetric quantum mechanics were used to study the systems with smooth vector potential [33,34], electrostatic potential [35] or their combination [36]. (Quasi-) exactly solvable systems associated with the solutions of Heun equation were discussed in [37].

In the current article, we focus on the qualitative analysis of the quantum transport in the magnetic waveguides. We aim at the systems where explicit solutions of the stationary equation do not exist analytically. Our goal is to provide as detailed as possible information on the spectrum of the considered systems. In particular, we will be interested in the existence and properties of the energy bands that are the hallmark of dispersionless wave packets in the systems with translational invariance. Our analysis is based on the variational principle and asymptotic behavior of the vector potential. We will analyze in detail realistic settings where magnetic field is generated either by a set of electric wires or magnetized strips with different direction of magnetization, posed in the proximity of the two-dimensional Dirac fermions.

The work is organized as follows. In the next section, we rigorously define the considered model and explore its spectral properties in dependence on the asymptotic behavior of the vector potential. We review existing relevant results and introduce the new tools for spectral analysis in the form of statements together with their proofs. In the next section, we focus on the bundles of electric wires with zero net-current. Our motivation is two-fold there. First, the results should contribute