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Deformed quantum double realization of the toric code and beyond

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ABSTRACT

Quantum double models, such as the toric code, can be constructed from transfer matrices of lattice gauge theories with discrete gauge groups and parametrized by the center of the gauge group algebra and its dual. For general choices of these parameters the transfer matrix contains operators acting on links which can also be thought of as perturbations to the quantum double model driving it out of its topological phase and destroying the exact solvability of the quantum double model. We modify these transfer matrices with perturbations and extract exactly solvable models which remain in a quantum phase, thus nullifying the effect of the perturbation. The algebra of the modified vertex and plaquette operators now obey a deformed version of the quantum double algebra. The Abelian cases are shown to be in the quantum double phase whereas the non-Abelian phases are shown to be in a modified phase of the corresponding quantum double phase. These are illustrated with the groups $\mathbb{Z}_n$ and S_3. The quantum phases are determined by studying the excitations of these systems namely their fusion rules and the statistics. We then go further to construct a transfer matrix which contains the other $\mathbb{Z}_2$ phase namely the double semion phase. More generally for other discrete groups these transfer matrices contain the twisted quantum double models. These transfer matrices can be thought of as being obtained by introducing extra parameters into the transfer matrix of lattice gauge theories. These parameters are central elements belonging to the tensor products of the algebra and its dual and are associated to vertices and volumes of the

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three dimensional lattice. As in the case of the lattice gauge theories we construct the operators creating the excitations in this case and study their braiding and fusion properties.

1. Introduction

The concept of topological order [1] was initiated in the 80s with the discovery of the fractional quantum Hall effect and high temperature superconductivity [2–4]. Since then it has also been seen in short range resonating valence bond states [5–8] and in quantum spin liquids [9–16]. Once its usefulness was realized in solid state quantum computation [17] several exactly solvable models have been constructed achieving this. The classic example emerged when Kitaev wrote down the toric code Hamiltonian in two dimensions [18]. These systems are examples of lattice models which host anyons [19] as part of their low energy excitations. They can also be thought of as particular phases of the $\mathbb{Z}_2$ lattice gauge theory which host these deconfined excitations with anyonic statistics. They possess ground states with degeneracies which are topological invariants. This degeneracy is stable up to the addition of weak perturbations. This feature makes this model a probable candidate for fault tolerant quantum computation [18]. However we must point out that there are more proposals for fault tolerant quantum computation as introduced in [20,21] and also recently, a minimal instance of these models have been realized experimentally [22].

The quantum double models of Kitaev have been extended to other discrete groups and involutory Hopf algebras as well [18,23]. In these cases they can be thought of as arising from lattice gauge theories based on these discrete groups or involutory Hopf algebras [24]. Earlier works showing the existence of anyons in two dimensional discrete gauge theories can be found in [25,26]. Discrete gauge theories emerge in these models when a continuous gauge group breaks down to one via spontaneous symmetry breaking [27–33].

These systems are usually perturbed by adding qudit terms to act on the edges of the lattice which carry the gauge degrees of freedom. They drive the system out of the topological phase [34–40]. The resulting models are rendered unsolvable analytically and are thus subject to study using numerical methods. However by considering restricted plaquette and vertex operators they can be made solvable. Such studies were carried out in [41]. This resulted in condensed phases of the quantum double model. These works were crucial in understanding the stability of these topological phase represented by the quantum double model. It is thus an important problem to find exactly solvable models which remain in topological phases in the presence of these perturbations.

In this spirit we introduce exactly solvable models which are constructed by looking at possible Hamiltonians that can be generated using the transfer matrices of such systems. We show exactly solvable models which include the single qudit perturbations. The cases of Abelian and non-Abelian groups are studied separately. It is shown that in the Abelian case the system remains in the topological phase corresponding to the quantum double model. The situation turns out different for the non-Abelian cases. We find that the model remains in a topological phase but it is in a modified version with respect to the corresponding topological phase of the quantum double model. These are seen by studying the examples of the group algebras of $\mathbb{Z}_n$ and S_3 denoted by $\mathbb{C}(\mathbb{Z}_n)$ and $\mathbb{C}(S_3)$ respectively.

We then go beyond the transfer matrices of lattice gauge theories by introducing more parameters in the transfer matrix of lattice gauge theories to find other topological phases. This transfer matrix is made up of two-qudit operators apart from the usual operators making up the transfer matrix of lattice gauge theories. We work with the $\mathbb{C}(\mathbb{Z}_2)$ case to show how one can obtain the double semion phase from such considerations. For more general groups these transfer matrices contain the twisted quantum double models as defined in [42]. Such models were also defined in [43] while considering trivial global symmetry groups.