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Behavior of Werner states under relativistic boosts

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ABSTRACT

We study the structure of maps that Lorentz boosts induce on the spin degree of freedom of a system consisting of two massive spin-1/2 particles. We consider the case where the spin state is described by the Werner state and the momenta are discrete. Transformations on the spins are systematically investigated in various boost scenarios by calculating the orbit and concurrence of the bipartite spin state with different kinds of product and entangled momenta. We confirm the general conclusion that Lorentz boosts cause non-trivial behavior of bipartite spin entanglement. Visualization of the evolution of the spin state is shown to be valuable in explaining the pattern of concurrence. The idealized model provides a basis of explanation in terms of which phenomena in systems involving continuous momenta can be understood.

1. Introduction

Entanglement is widely regarded as one of the central features that sets the quantum and classical worlds apart. Motivated by its fundamental importance as well as promises of application, the theory of entanglement has made vast progress over the last decades. Recently there has been growing interest in relativistic quantum information. This takes seriously the notion that the ultimate description of physical reality is relativistic and seeks to provide an account of how the quantum information theoretic notions like entanglement behave in the relativistic regime.

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Extensive research on both single and two particle systems has uncovered a wealth of results about how relativity affects entanglement [1–17]. Early work found that spin entanglement of a bipartite system does not remain invariant under Lorentz boosts [3]. This was confirmed by [2] who reported that spin entropy of a single particle is not a relativistic scalar. On the other hand, [6] argued that the entanglement fidelity of a Bell state remains invariant for a Lorentz boosted observer. Further research found that no sum of entanglements remains invariant under boosts [18].

A key aspect one notices is that many of these sometimes seemingly conflicting results involve systems containing different momentum states and boost geometries. This confirms what has been observed in single particle systems: entanglement under Lorentz boosts is highly dependent on the boost scenario in question [17]. While the literature on the Wigner rotation is quite clear about the fact that its nature is highly geometric, aside from a few cases [19], there is little work in relativistic quantum information that systematically takes this into account.

This paper draws motivation from both mentioned aspects. In light of the diverse results it is not immediately clear what one should infer about entanglement in relativity. For instance, is it the case that it remains invariant or not? What are the conditions under which either occurs? The question arises if there is a systematic way to understand such behavior. The nature of the Wigner rotation implies that one should ask what is role of the geometry of the underlying physical situation in determining the behavior of entanglement.

To address these queries we set out to explore entanglement in a number of boost scenarios with different momenta as well as geometries. We focus on massive two particle spin-1/2 systems with discrete momenta in product and entangled states. While it is common to assume that the spin state is given by a maximally entangled Bell state, we extend the treatment to mixed states by considering spins in the Werner state. This enables us to study how different classes of entanglement, ranging from maximal to zero, behave in relativity. We also discuss how one can visualize the orbit of the spin state in a 3D manner in order to gain deeper insight into how entanglement changes under Lorentz boosts. The aim is to provide a simple discrete framework which can be used to explain the results involving both discrete as well as narrow continuous momenta. Surveying a range of different momenta and geometries will also contribute to an overview of the kinds of systems that could be of interest for relativistic quantum information.

The paper is organized as follows. We begin by setting the stage in Section 2, followed by a characterization of the Thomas–Wigner rotation. The next three sections describe the discrete model used throughout the paper, focusing on the momentum and spin states in Sections 5 and 6, respectively. Thereafter we turn to studying the behavior of mixed spin states in boost scenarios which contain different kinds of product and entangled momenta. We conclude with a discussion of the results obtained.

2. The general setting

We will focus on a system consisting of two massive spin-1/2 particles with spin and momentum, and ask how the spin state changes when viewed from a different inertial frame. This question has a trivial answer in non-relativistic quantum theory: the state will remain unchanged. But the relativistic world is different. A Lorentz boosted observer will see in general a transformed spin state and the reason is the so-called Wigner rotation, or Thomas–Wigner rotation (TWR), where the latter form is commonly used in honor of Thomas' contribution by discovering the Thomas precession [20–22]. By way of illustration, consider a simple, one particle system which forms the smallest entity—the 'qubit'—of relativistic quantum information in inertial frames. Suppose the particle is moving relative to observer O who describes its state by

$$|\Psi\rangle = \sum_{\mathbf{p},\lambda} \psi_\lambda(\mathbf{p}) \, |\mathbf{p}, \lambda\rangle \,, \tag{1}$$

where $|\mathbf{p}, \lambda\rangle \equiv |\mathbf{p}\rangle \, |\lambda\rangle \equiv |\mathbf{p}\rangle \otimes |\lambda\rangle$ is a basis vector with $\mathbf{p}$ labeling the momentum and λ the spin of the particle, see Appendix for details of the constructions used in the paper. For the sake of illustration, let us restrict our attention to discrete momentum states for now. Observer O^Λ who is Lorentz boosted by Λ relative to O assigns in general a different state $\left|\Psi^\Lambda\right\rangle = U(\Lambda)\,|\Psi\rangle$ to the same system, where