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Discussion

An explicit analytical expression for bed-load layer thickness based on maximum entropy principle

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ABSTRACT

The present study aims to derive an analytical model on bed-load layer thickness in an open channel turbulent flow carrying sediments. Determination of the thickness of the bed-load layer is of utmost importance in the study of bed-load transport as it is required to determine the bed-load transport rate, as well as in the study of suspended load transport as it acts as reference level for the particles in suspension. Apart from the several deterministic approaches available in the literature, the work adopts probabilistic approach based on entropy theory to determine the bed-load layer thickness. The concept of entropy theory developed by Shannon is used and the method of Lagrange multipliers is employed for the maximization of entropy function to find the least biased probability distribution. To calculate the Lagrange multipliers, present in the probabilistic model of dimensionless bed-load layer thickness, two different methodologies are presented. The model of bed-load layer thickness is a function of dimensionless shear stress and also depends on three other parameters which are found to be functions of specific gravity of sediment particle and dimensionless particle diameter from a non-linear regression analysis. The proposed model is validated with wide sets of experimental data available in literature and a good agreement is achieved. Apart from comparison with data, the model is also compared with existing deterministic model and computation of relative percentage error proves the better efficiency of the present model.

1. Introduction

In the field of hydrodynamics and hydraulics, the mechanism of transportation of non-cohesive sediments in a turbulent flow is a fundamental topic of research. Particles are transported in the flow by two different modes: bed-load and suspended load [16]. Near to the bed, the movements of particles occur by sliding, rolling and jumping or saltation, and are carried out by the main flow as bed-load. The saltation height of a particle defines the bed-load layer thickness (see Fig. 1), usually denoted by δ, determination of which is a matter of keen interest to the researchers as it plays an important role in the sediment transport mechanism [14], [11].

Several researchers attempted to develop theories and expressions to address the thickness of the bed-load layer in a flow carrying sediments. Einstein [13] defined it as two grain diameter thick which is easy to deal with. Van Rijn [50] proposed an expression for δ, which is a function of particle diameter and flow transport capacity. Wilson [53] analytically derived a model for δ which is a simple expression of non-dimensional shear stress whereas the bed-load layer thickness developed by Wiberg and Rubin [52] is related to the bed roughness. Apart from theoretical study, saltation was studied experimentally by several researchers (Devries [12], Nino and Garcia [41], Nino et al. [42]). Based on hydrodynamic diffusion concept related to particle–particle interactions, Cheng [11] proposed an analytical model on δ. Saltating process was also studied for single and multiple sediment particles in both two and three dimensions by several researchers (Kharlamova and Vlasak [25], Lee et al. [35], [34], Wang et al. [51]). It was also studied by video imaging with varying bed roughness by Bhattacharyya et al. [1]. Bialik et al. [2] studied the influence of turbulence on saltating grains through numerical study. Including different factors of turbulence such as particle–particle collisions, particle diffusion and others, three dimensional numerical model of saltation trajectories for a spherical particle was studied by different researchers (Bialik et al. [3], Lukerchenko et al. [36], [37], Moreno and Bombardelli [40]).

All the above mentioned works reflect clearly that the bed-load layer thickness has been studied only through deterministic approaches so far. But any study in turbulence is always associated

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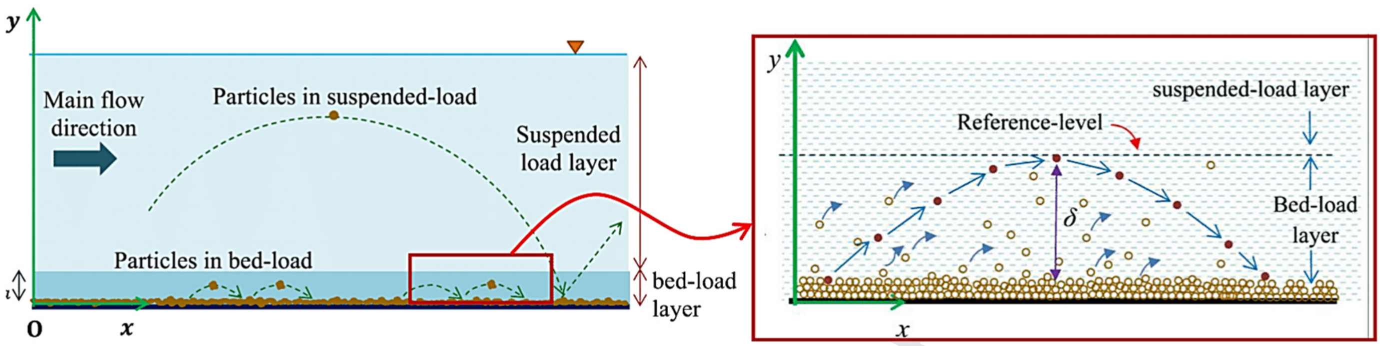

Fig. 1. Schematic diagram of the bed-load layer thickness.

with uncertainties in variables and model parameters involved in the study that may not be measured by deterministic approaches. These approaches make use of the conservational laws in fluid dynamics such as equations of continuity, momentum and energy which often lead to the situation that the number of unknowns exceeds the number of equations involved in the derivation, and hence empirical or semi-empirical equations are often needed in order to find the solution. However, the shortcomings of deterministic approaches may perhaps be addressed through probabilistic treatment based on entropy theory. Entropy measures the uncertainties within a system and acts as a connection between deterministic and probabilistic world [5]. Most fundamental laws in physics and mechanics can also be developed from the theories of entropy [54]. The concept of entropy and the maximum entropy principle have been successfully applied to a wide range of science and engineering problems since long [5], [55], [17], [43]. For the last two decades, several fluvial hydraulics problems have been given a probabilistic treatment through entropy theory and have been solved successfully. The concept of entropy due to Shannon [45] along with Jaynes' [21], [22] principle of maximum entropy (POME) has been applied to predict the spatial distribution of velocity [5], [6], [7], [8], [10], [47], sediment concentration [9], [27], [28], [30], [31], shear stress [48], the hindered settling velocity [29], position of maximum velocity in a narrow open channel [32], [33] and few others [4], [26], [39]. In this type of problem of prediction, the maximization of entropy is not used as a law of physics, but as a method of reasoning that does not consider any unconscious arbitrary assumptions [21], [22]. The procedure of this approach entails the steps: (i) First, define the entropy and the available information, (ii) then define the given information in terms of constraints, (iii) maximize the entropy subject to those defined constraints, (iv) derive the least-biased probability distribution, and finally (v) determine the distribution parameters in terms of constraints.

The objective of the present work is to extend the applicability of entropy theory through a study on the thickness of bed-load layer in a turbulent flow carrying sediments. The derivation is started from the definition of Shannon entropy and the constraints are considered by satisfying the total probability law and the mean constraint, as usual. The entropy function along with the constraints is maximized in accordance with POME, and the Lagrange multipliers present in the derived most probable probability distribution are computed by two different ways, namely, (i) algebraic method, and (ii) method of maximum likelihood estimator (MLE). The expression for bed-load layer thickness, obtained through the hypothesization of a non-linear cumulative distribution function, is compared with experimental measurement available in literature and quite a good agreement is observed which confirms the applicability of entropy theory in the context of studying fluvial processes.

2. Mathematical formulation of the problem

A schematic diagram of saltation process in sediment-laden flow of depth h is shown in Fig. 1. The saltation height, which is the thickness of bed-load layer, is denoted as δ. The present study assumes the dimensionless bed-load layer thickness $\hat{\delta}$, i.e., the ratio of δ to D, D being the particle diameter, as a random variable. Our objective is to derive an entropy-based analytical expression of $\hat{\delta}$. To that end, the derivation using the concept of Shannon entropy together with POME entails the steps: (1) definition of Shannon entropy, (2) principle of maximum entropy, (3) constraint equations, (4) maximization of entropy, (5) hypothesis on cumulative distribution function (CDF), (6) determination of bed-load layer thickness, and (7) determination of Lagrange multipliers. Each of these steps is discussed in what follows.

2.1. Definition of Shannon entropy

The concept of information entropy was introduced by Claude Shannon [45]. The concept he developed was based on measure of uncertainty or information of a probability distribution or random variable associated with a system. For arbitrary uncertain systems, let X be a random variable to represent the system state features, and n is the number of values that the random variable takes on; p_j's are the probabilities of the random variable for $j = 1, 2, \dots, m$. Then, the Shannon entropy [45] of X can be written as

$$H_S(X) = -\sum_{j=1}^{m} p_j \ln p_j \tag{1}$$

Eq. (1) expresses a measure of uncertainty about p_j's or the average information content in the sampled p_j's. Theoretically, entropy function given in Eq. (1) is maximum when all the p_j's are equiprobable, i.e., probabilities are uniform within its limits. Analogous to Eq. (1), Shannon entropy in continuous form can be written as

$$H_S(X) = -\int_{\hat{\delta}_{min}}^{\hat{\delta}_{max}} f(\hat{\delta})\left[\ln f(\hat{\delta})\right] d\hat{\delta} \tag{2}$$

where $f(\hat{\delta})$ is the probability density function, $\hat{\delta}_{min}$ and $\hat{\delta}_{max}$ are the lower and upper limits of the random variable $\hat{\delta}$, respectively. In a strict sense, Eq. (2) cannot represent uncertainty because it can also produce negative values and is not invariant under coordinate transformation [24]. However, the definition does not create any problem while using the POME [21], [22], [23], because the objective of this study is to maximize the entropy in order to get the least biased probability density function $f(\hat{\delta})$.