Physics Letters A 382 (2018) 2245–2250

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

# Dielectric and magnetic properties of continuum with dislocations

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ARTICLE INFO

*Article history:*
Available online 14 September 2017
Communicated by S. Vyatchanin

*Keywords:*
Solid
Electrodynamics
Dislocations
Permittivity
Permeability
Refractive index

ABSTRACT

More than twenty years ago, we were invited by professor V.B. Braginsky at the seminar of the Molecular Physics and Physical Measurements Department to make a report on using gauge fields, fibre bundles, and geometry to describe solids with structural defects. After the seminar, he proposed us to consider a possibility of using the above-mentioned exact mathematical and geometrical methods to create theoretical models of description of optomechanical properties of media with defects. It was the starting point for us to investigate the electromagnetic wave propagation in solids with defects. This paper is dedicated to the memory of professor Vladimir Borisovich Braginsky. The solutions of the electrostatics and magnetostatics equations for a continuous medium with dislocations are found. The expressions for the dependence of the electric permittivity and magnetic permeability on the dislocation distribution in a solid are found. The expression for the refractive index in a continuum with dislocations is obtained.


## 0. Introduction

All real crystals contain structural defects. Lattice vacancies, interstitial atoms, and their numerous complexes are formed in the process of crystal growing, under plastic deformations, radiation exposure and other external effects. Dislocations are a natural product of crystallization and material treatment. In fact, real crystals virtually do not contain isolated dislocation lines. Besides, point, linear, and planar defects can move, form strongly interacting ensembles, be generated at dislocation sources, disappear at sinks and undergo mutually transformations. For example, there are dislocation clusters in the form of coiled balls, plain nets or dipole structures. Regular groups of dislocations form disclinations and various kinds of boundaries of cells, blocks, fragments, and grains. The boundaries of disorientation can be both the sources and the sinks of linear and point defects.

The presence of structural defects strongly influences many properties of crystals. Thus, such fundamental characteristics of crystals as their strength and plasticity are determined almost exclusively by their defect structure. For example, the yield point for a crystal grown with observing usual precautions can be equal to about $1,0$ MPa, while so-called crystal "whiskers", which are virtually ideal crystals, have the yield point equal to about $0,1 \cdot E \sim 10^4$ MPa. On the other hand, a similar value of the yield point is characteristic for strongly cold-hardened metals. The wire made of molybdenum–rhenium alloy, which underwent a drawing elongation of $6 \cdot 10^6\%$ at 293 $K$, does not fail at the strain up to 9500 MPa [1,2]. It should be noted that mechanical properties of amorphous metals also depend on the peculiarities of the irregular ("defect") arrangement of atoms in them. It is clear from all mentioned above that a physical model, which would adequately describe macroscopic properties of crystals, should be based on the concept of a crystal as of a medium with many structural defects. Therefore, physics of structural defects is traditionally one of the most important branches of condensed matter physics.

Unfortunately, there is no appropriate theoretical description of real crystals that would be applicable to theoretical investigations of mechanical, optical, and other physical properties of real crystals. An appropriate description of an ideal crystal in the investigations of physical phenomena with a characteristic length greater than the crystal lattice parameter can be achieved by describing an ideal crystal as a medium that is the Euclidean manifold. In other words, it is possible to apply the continuous approximation of discrete matter to describe such phenomena. Hence, when analyzing the processes of plastic deformation and destruction of real materials, one can ignore the peculiarities of dislocations and consider their continuous distribution in a certain area. From this point of view, crystals with defects can be represented as the Riemannian and non-Riemannian manifolds.

In terms of the gauge description of structural defects in solids [3–9], it is shown that a continuous medium with defects can be modeled as the Riemann–Cartan manifold $\mathcal{U}_4$ with non-Euclidean

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http://dx.doi.org/10.1016/j.physleta.2017.09.015
0375-9601/

metric $g_{\mu\nu}$ and non-symmetric connection $\Gamma^{\lambda}_{\mu\nu}$. Besides, the internal stresses in crystals caused by the presence of lattice defects are modeled as a change in the continuum geometry. In such a case, dislocations correspond to the presence of non-zero components in the torsion tensor $Q^{\alpha}_{\mu\nu}$ [5,9]. In this study, we will not go beyond the case when the influence of external stresses on the movement of dislocations inside the solid (deformations are small or absent) may be neglected, and we will assume that the manifold metric modeling the solid coincides with the Euclidean one, and the connection is given only by its spatial components: $\Gamma^{i}_{jk} = Q^{i}_{jk} + Q^{i}_{jk} + Q^{i}_{kj} = \Gamma_{ijk}$. We will also assume that the distribution of structural defects is stationary and its change caused by the interactions with external fields may be neglected. Such an approach will allow us to study the influence of dislocations on electromagnetic processes inside of solids containing linear defects from the macroscopic point of view, i.e. in the case when the characteristic length of a physical process exceeds the values of the lattice parameters.

In section 1, we will remind the principal ideas of the electrodynamics of continuous media with defects described in papers [10,11]. In section 2, we will find the solutions to the electrostatics and magnetostatics equations for the case of the antisymmetric tensor of dislocation density. In section 3, we will investigate the dependence of the tensors of electric permittivity and magnetic permeability of a continuous medium containing defects on the edge dislocation distribution density tensor. In the Conclusions, we will analyze the obtained results and prospects for further investigations.

## 1. Electromagnetic field in a medium with defects

In paper [10], it was shown that the experimentally determined dislocation density tensor $\hat{\rho}$ can be related to the torsion tensor $\hat{Q}$ equal to

$$Q^{i}_{kl} = \varkappa \epsilon_{jkl} \rho^{ij}, \tag{1}$$

where $\epsilon_{jkl}$ is the antisymmetric Levi-Civita symbol, $\varkappa$ is a constant (the constant of interaction between electromagnetic field and defects) which agrees with [3,12]. Since we consider only the purely dislocation free state and the metric $g_{ij}$ coincides with the Euclidean metric $\delta_{ij}$, we will not distinguish lower and upper indices. The usual rule of summation over doubly repeated indices is supposed.

In this case, the equations for the electromagnetic field in a continuous medium with a stationary dislocation distribution are following [10]:

$$\begin{aligned}
&\mathrm{div}\vec{D} = \varkappa \left( (\hat{\rho})_{ij} - (\hat{\rho}^{\top})_{ij} \right) \epsilon_{ikj} D_k, \\
&\frac{1}{c} \cdot \frac{\partial \vec{D}}{\partial t} - \mathrm{curl}\vec{H} = 2\varkappa \hat{\rho}^{\top} \vec{H}, \\
&\mathrm{div}\vec{B} = \varkappa ((\hat{\rho})_{ij} - (\hat{\rho}^{\top})_{ij}) \epsilon_{ikj} B_k, \\
&\frac{1}{c} \cdot \frac{\partial \vec{B}}{\partial t} + \mathrm{curl}\vec{E} = -2\varkappa \hat{\rho}^{\top} \vec{E},
\end{aligned} \tag{2}$$

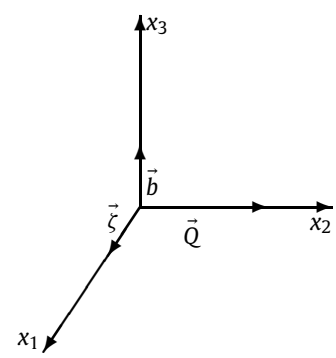
where $\vec{E}$ is the electric field strength, $\vec{D}$ is the electric displacement field, $\vec{H}$ is the magnetic field strength, $\vec{B}$ is the magnetic field, $(\hat{\rho}^{\top})_{ij} = (\hat{\rho})_{ji}$ is the transposed matrix of the dislocation density tensor. Here and in what follows, the designation $\hat{\rho}^{\top}\vec{E}$ stands for $(\hat{\rho}^{\top}\vec{E})_i = (\hat{\rho}^{\top})_{ij} E_j$ and so on.

Let us transform the equation system (2) to a more simple form for the case of the antisymmetric dislocation density tensor ($\rho_{ij} = -\rho_{ji}$). At first, we will find a relation between the trace of the contortion tensor and the dislocation density tensor:

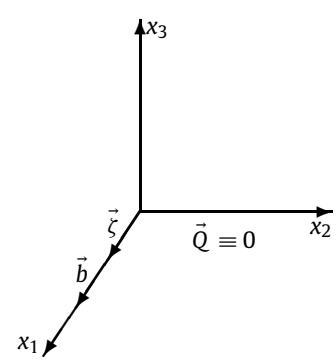
$$K_k = K_{iki} = Q_{iki} + Q_{kii} + Q_{iki} = 2\kappa \epsilon_{kil} \rho^{il}. \tag{3}$$

Let us introduce the vector $\vec{Q}$ equal to the trace of the contortion tensor multiplied by $-1$, i.e. $(\vec{Q})_i = -(\vec{K})_i$.

As an illustration, let us consider two simplest cases. The first example is a single edge dislocation with the dislocation line $\vec{\zeta}$ collinear to the $x_1$ axis and the Burgers vector $\vec{b}$ collinear to the $x_3$ axis. In this case, the components of the dislocation density tensor are equal to $\rho_{ij} = \delta_{i1}\delta_{j3} b\delta(r)$, where $\delta(r)$ is the delta-function depending on the distance to the dislocation line. The dislocation density tensor has the form $\hat{\rho} = \begin{pmatrix} 0 & 0 & b\delta(r) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Let us calculate the components of the vector $\vec{Q}$ for this case: $Q_i = -2\varkappa \varepsilon_{ijl} \rho_{jl} = -2\varkappa b \varepsilon_{i13} \delta(r)$. Hence, $\vec{Q} = (0, 2\varkappa b\delta(r), 0)$.




The second example is a single screw dislocation with both the dislocation line $\vec{\zeta}$ and the Burgers vector $\vec{b}$ collinear to the $x_1$ axis. In this case, the components of the dislocation density tensor are equal to $\rho_{ij} = \delta_{i1}\delta_{j1} b\delta(r)$, therefore, the dislocation density tensor $\hat{\rho} = \begin{pmatrix} b\delta(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Let us calculate the components of the vector $\vec{Q}$ for this case: $Q_i = -2\varkappa \varepsilon_{ijl} \rho_{jl} \equiv 0$, since the dislocation density tensor for the screw dislocations is symmetric.




Let us transform equation system (2) to a simpler form in the case of the antisymmetric dislocation density tensor ($\rho_{ij} = -\rho_{ji}$). Then the first equation of the system can be written as follows:

$$\begin{aligned}
\mathrm{div}\vec{D} = \varkappa \left( (\hat{\rho})_{ij} - (\hat{\rho}^{\top})_{ij} \right) \epsilon_{ikj} D_k = \\
- 2\varkappa \rho^{(a)}_{ij} \epsilon_{kij} D_k = D_k \left( -2\varkappa \epsilon_{kij} \rho^{(a)}_{ij} \right) = \\
D_k (-2) Q_{iki} = D_k (-\vec{K})_k = \vec{Q} \cdot \vec{D}.
\end{aligned}$$