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Relativistic entanglement

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ABSTRACT

The relativistic quantum theory of Stueckelberg, Horwitz and Piron (SHP) describes in a simple way the experiment on interference in time of an electron emitted by femtosecond laser pulses carried out by Lindner et al. In this paper, we show that, in a way similar to our study of the Lindner et al. experiment (with some additional discussion of the covariant quantum mechanical description of spin and angular momentum), the experiment proposed by Palacios et al. to demonstrate entanglement of a two electron state, where the electrons are separated in time of emission, has a consistent interpretation in terms of the SHP theory. We find, after a simple calculation, results in essential agreement with those of Palacios et al.; but with the observed times as values of proper quantum observables.

1. Introduction

Palacios, Rescigno and McCurdy [1] have described a proposed experiment which could show entanglement of a two electron system in which each electron is emitted at a slightly different time. Although the anticipation of this effect is very reasonable, it does not have a theoretical justification in the framework of the standard nonrelativistic quantum theory, since in the nonrelativistic theory, both electrons must be prepared in states at precisely equal times. As for the Lindner et al. [2] experiment showing interference in time for the wave function of a particle, for which extensive calculations were done using the nonrelativistic Schrödinger evolution of the electron, wave functions at different times (corresponding to elements of different Hilbert spaces [3]) are incoherent in the nonrelativistic quantum theory. The direct product states corresponding to the basis for many body systems must, in the same way, be constructed from states in the same Hilbert space. Therefore, the same conclusion can be reached for the entanglement of the spins of a two body system. In actual practice, in fact, it would not be possible experimentally to generate two body states at precisely equal times, so that it is important to construct a theoretical basis, as we shall do below, in which effects of the type we expect to see (and are seen, for example, in the experiment of Lindner et al. [2]) can be consistently described.

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The nonrelativistic theory of the two body state with spin is constructed from linear combinations of direct product wave functions taken at equal time [4]. One could argue intuitively from the vector model, in which the result $\mathbf{J}^2 = j(j+1)$ (for $\mathbf{J}$ the angular momentum operator, and j the integer or half-integer eigenvalue), that it appears that the physical angular momentum is not precisely along the "direction" of the vector $\mathbf{J}$, but can be thought of as precessing around it. The entangled spin zero state of two spin $1/2$ systems therefore would be the result of an exact synchronization of these oppositely oriented precessing spins so that the total angular momentum is zero. At slightly different times, this synchronization would be, in principle, lost. Under nonrelativistic Schrödinger evolution the superposition of two-body states at different times would therefore be ineffective. Stated more rigorously, states are not coherent [3] at nonequal times and linear superposition is not defined in the nonrelativistic theory.

As for the Lindner et al. experiment [2], an explanation can be given in terms of the relativistic quantum theory of Stueckelberg, Horwitz and Piron (to be called SHP) [5]. The computation in terms of the SHP [6] was in precise agreement with the experimental result (actually predicted in 1976 [7], when the technology was not available for verification). In this paper, we apply a similar reasoning to the entangled two body state.

We start with a review of the basic SHP theory [5] and a discussion of how the Wigner theory of induced representations for relativistic spin is applied in this framework. We then argue that the proposed experiment of Palacios et al. should yield well-defined entanglement for the constituent particles at not precisely equal times.

Stueckelberg [5], in 1941, imagined that a particle world line would be straight for no interaction, but that interaction could bend the world line so that it would turn to propagate in the negative direction of time. To describe such a picture, he introduced an invariant parameter along the world line, which he called τ, and interpreted the backward in time evolving branch of the line as an antiparticle. Horwitz and Piron [5] then generalized this idea in the sense that the parameter τ was to be considered as a universal invariant time, as for the original postulate of Newton, in order to formulate the many body problem in this framework, as we discuss below.

As a model for the structure of the dynamical laws that might be considered, Stueckelberg proposed a Lorentz invariant Hamiltonian for free motion of the form

$$K = \frac{p^\mu p_\mu}{2M}, \tag{1.1}$$

where M is considered a parameter, with dimension mass, associated with the particle being described, but is not necessarily its measured mass. In fact, the numerator (with metric $-+++$; we generally take $c=1$),

$$p^\mu p_\mu = -m^2, \tag{1.2}$$

corresponds to the actual observed mass (according to the Einstein relation $E^2 = \mathbf{p}^2 + m^2$), where, in this context, m^2 is a dynamical variable.

The Hamilton equations, generalized covariantly to four dimensions, are then

$$\begin{aligned} \dot{x}^\mu &\equiv \frac{dx^\mu}{d\tau} = \frac{\partial K}{\partial p_\mu} \\ \dot{p}_\mu &\equiv \frac{dp_\mu}{d\tau} = -\frac{\partial K}{\partial x^\mu}. \end{aligned} \tag{1.3}$$

These equations are postulated to hold for any Hamiltonian model, such as with additive potentials or gauge fields. A Poisson bracket may be then defined in the same way as for the nonrelativistic theory. The construction is as follows. Consider the τ derivative of a function $F(x,p)$, *i.e.*,

$$\begin{aligned} \frac{dF}{d\tau} &= \frac{\partial F}{\partial x^\mu}\frac{dx^\mu}{d\tau} + \frac{\partial F}{\partial p^\mu}\frac{dp^\mu}{d\tau} \\ &= \frac{\partial F}{\partial x^\mu}\frac{\partial K}{\partial p_\mu} - \frac{\partial F}{\partial p^\mu}\frac{\partial K}{\partial x_\mu} \\ &= \{F, K\}, \end{aligned} \tag{1.4}$$

thus defining a Poisson bracket $\{F, G\}$ quite generally. The arguments of the nonrelativistic theory then apply, *i.e.*, that functions which obey the Poisson algebra isomorphic to their group algebras will have vanishing Poisson bracket with the Hamiltonian which has the symmetry of that group, and are thus conserved quantities, and the (τ independent) Hamiltonian itself is then (identically) a conserved quantity.

It follows from the Hamilton equations that for the free particle case

$$\dot{x}^\mu = \frac{p^\mu}{M} \tag{1.5}$$

and therefore, dividing the space components by the time components, cancelling the $d\tau$'s ($p^0 = E$ and $x^0 = t$),

$$\frac{d\mathbf{x}}{dt} = \frac{\mathbf{p}}{E}, \tag{1.6}$$

the Einstein relation for the observed velocity. Furthermore, we see that

$$\dot{x}^\mu \dot{x}_\mu = \frac{p^\mu p_\mu}{M^2}; \tag{1.7}$$

with the definition of the invariant

$$ds^2 = -dx^\mu dx_\mu, \tag{1.8}$$

corresponding to proper time squared (for a timelike interval), this becomes

$$\frac{ds^2}{d\tau^2} = \frac{m^2}{M^2}. \tag{1.9}$$

Therefore, the proper time interval Δs of a particle along a trajectory parametrized by τ is equal to the corresponding interval $\Delta\tau$ only if $m^2 = M^2$, a condition we shall call "on mass shell".

Stueckelberg [5] formulated the quantized version of this theory by postulating the commutation relations

$$[x^\mu, p^\nu] = i\hbar g^{\mu\nu}, \tag{1.10}$$

where $g^{\mu\nu}$ is the Lorentz metric given above, and a Schrödinger type equation (we shall take $\hbar = 1$ in the following)

$$i\frac{\partial}{\partial\tau}\psi_\tau(x) = K\psi_\tau(x), \tag{1.11}$$

where $\psi(x)$ is an element of a Hilbert space on R^4 satisfying

$$\int |\psi(x)|^2 d^4x = 1, \tag{1.12}$$

and satisfies the required Hilbert space property of linear superposition. With the generalization of Horwitz and Piron [5], Eq. (1.11) can be written for any number N of particles as

$$i\frac{\partial}{\partial\tau}\psi_\tau(x_1, x_2 \ldots x_N)) = K\psi_\tau(x_1, x_2 \ldots x_N), \tag{1.13}$$

where K could have, for example, the form

$$K = \Sigma_i^N \frac{p_i{}^\mu p_{i\mu}}{2M_i} + V(x_1, x_2 \ldots x_N), \tag{1.14}$$

and $V(x_1, x_2 \ldots x_N)$ is assumed, for our present purposes, to be Poincaré invariant.

The basis of the Hilbert space describing such states is provided by the direct product of one particle wave functions *taken at equal* τ (as for equal time t in the nonrelativistic theory [4]). In the following, we apply this structure to the description of two particles with spin.

2. Relativistic spin and the Dirac representation

We shall discuss in this section the basic idea of a relativistic particle with spin, based on Wigner's seminal work [8]. The theory is adapted here to be applicable to relativistic quantum theory; in this form, Wigner's theory, together with the requirements imposed by the observed correlation between spin and statistics in nature for identical particle systems, makes it possible to define the total spin of a state of a relativistic many body system.

The spin of a particle in a nonrelativistic framework corresponds to the lowest dimensional nontrivial representation of the rotation group; the generators are the Pauli matrices σ_i divided by two, the generators of the fundamental representation of the double covering of $SO(3)$. The self-adjoint operators that are the generators of this group measure angular momentum and are associated with magnetic moments. Such a description is not relativistically covariant, but Wigner [8] has shown how to describe this