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Theories and applications of second-order correlation of longitudinal velocity increments at three points in isotropic turbulence

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ABSTRACT

In order to introduce new physics to traditional two-point correlations, we define the second-order correlation of longitudinal velocity increments at three points and obtain the analytical expressions in isotropic turbulence. By introducing the Kolmogorov 4/5 law, this three-point correlation explicitly contains velocity second- and third-order moments, which correspond to energy and energy transfer respectively. The combination of them then shows additional information of non-equilibrium turbulence by comparing to two-point correlations. Moreover, this three-point correlation shows the underlying inconsistency between numerical interpolation and three-point scaling law in numerical calculations, and inspires a preliminary model to correct this problem in isotropic turbulence.

1. Introduction

One of the most important foundational researches in turbulence community is to find the multi-scale similarity. Specifically, this similarity is usually denoted by scaling laws of statistical quantities, which have indeed inspired and supported the understandings of various turbulent phenomena. In physical space, we list three different types of statistical quantities and summarize their relation to turbulent phenomena as follows:

1. *One-point statistical correlations* are usually related to energy production and dissipation. Consequently, they are widely employed in traditional turbulence models (such as the $k-\varepsilon$ model) in Reynolds averaged Navier–Stokes (RANS) calculations. An evident problem of the one-point statistical correlations is the lack of scale information.
2. *Two-point statistical correlations* involve an additional parameter of two-point distance, and thus provide the scale information. These statistical quantities are usually related to spatial energy transfer across scales. As a direct result, most scaling laws (such as the Kolmogorov 1941 scaling [1]) were introduced using two-point concepts. The two-point statistical correlations have all information in an Eulerian framework, but in the Lagrangian framework they are simply reduced to Lyapunov descriptions and lose the information of time direction for viscous flows.
3. *Four-point statistical correlations* were investigated in recent years due to their additional physics on time direction by comparing to the two-point statistical correlations [2–4]. Indeed, Lagrangian evolutions of four-point statistical correlations change not only the distances between points, but also the topologies via time-irreversible forms. These phenomena cannot be predicted simply using the traditional scaling laws.

Therefore, there is considerable interest in investigating what new understandings the *three-point statistical correlations* will involve. Reference [5] provided a very simple definition on the *three-point correlation* of passive scaler and revealed some interesting features about the structure of the scalar field, but investigations in velocity field still lack. Indeed, the lack of investigations on *three-point statistical correlations* in literature is probably due to the lack of corresponding new physics [6]. Accordingly, the non-equilibrium phenomena in turbulence, which have been discussed in recent dozen years [7–17], raise many new requirements of physical descriptions and emphasize the importance of *three-point statistical correlations*.

The traditional Kolmogorov 1941 theory (K41) [1] leads to two conclusions in the inertial range of high-Reynolds turbulent flows: *i*) $-5/3$ spectrum for the energy distribution; *ii*) transfer in the inertial range is always equilibrium to dissipation, corresponding to

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constant dissipation coefficient $C_\varepsilon := \langle\varepsilon\rangle L/\mathcal{U}^3$, with $\mathcal{U}$ root-mean-square (rms) of the velocity fluctuations, L integral length scale and $\langle\varepsilon\rangle$ turbulent kinetic energy dissipation rate. In recent years, a series of experimental and numerical evidences have shown that there exist in reality non-equilibrium turbulent flows, in which C_ε is not constant but rather inversely proportional to the Taylor-scale Reynolds number $C_\varepsilon \propto \mathrm{Re}_\lambda^{-1}$. Although having confusions with the non-equilibrium statistical mechanics, we follow literature and also describe this phenomenon as "non-equilibrium turbulence" [18]. Theoretical studies on the non-equilibrium turbulence are still in preliminary stage. Some references [16,19] consider the non-equilibrium part as a harmonic wave of the equilibrium energy spectrum with perturbations, which implies that the second-order moments change; by contrast, we explain the non-equilibrium turbulence as a local perturbation on third-order moments corresponding to energy transfer, which might lead to local departure to $-5/3$ energy spectrum in short time [14,15,17,20]. As an example for better understanding our explanation, changing all signs of velocities $\vec{u} \to -\vec{u}$ leads to an artificial extreme non-equilibrium field, where all even-order moments remain but odd-order moments change sign, yielding local imbalance between transfer and dissipation in inertial range. In physical space, this corresponds to the consideration of external perturbations in the phase space of velocity increments, and the slope of second-order structure function will then locally change according to Kolmogorov's equation, which is in agreement with the departure of $-5/3$ energy spectrum in spectral space. Although real non-equilibrium turbulent flows are not such extreme, they can be considered as the superposition between equilibrium and reversed fields [15]. This perturbation in phase space is also called as "phase de-coherence of turbulence" in the investigations of strong and weak turbulence, rotating turbulence and stratified turbulence, and cannot be described by two-point spatial correlation [21]. Therefore, the non-equilibrium phenomena contain information of the balance between energy distribution and energy transfer mechanism, which are related to different time scales. However, two-point statistical correlations are usually related to one time scale and cannot be used for measuring non-equilibrium quantities. From the view in physical space, the combination of energy and energy transfer must involve more information than two points. In this case, we investigate the properties of *three-point statistical correlations*, aiming at providing theoretical statistical tools for future studies on non-equilibrium turbulence.

In a previous study [22] we have discussed the endogenetic multi-scale correlations in homogeneous isotropic turbulence (HIT) by defining a second-order three-point correlation on velocity increments in a given direction. Two types of second-order three-point correlations, *i.e.*, the inertial-inertial type and the inertial-dissipative type, were discovered to be both endogenetic in HIT using this definition. This implies that the multi-scale correlation observed in wall-bounded turbulence [23] might also contain some endogenetic HIT correlations which should be clarified [22]. However, our ancient definition on three-point correlation in Ref. [22] is a mixture of longitudinal and transverse structure functions in most cases that the given direction is different from one of two vectors composed by the three point, and depends on the choose of coordinate system even in HIT. It is then interesting to ask what new interpretations and inspirations we could obtain by using a pure longitudinal definition to three points. For example, it is not clear whether the second-order three-point correlations have physical interpretations on non-equilibrium turbulence; in addition, from the practical view of numerical simulation, there are few discussions on the inconsistency between numerical interpolation and scaling laws.

In the present contribution we investigate a new definition of second-order three-point correlation on longitudinal velocity increments, aiming at providing new understandings for the above questions. In Sec. 2 we will present the relation between this three-point correlation and traditional two-point structure functions, and derive the analytical formula of the second-order three-point correlation on longitudinal velocity increments. In Sec. 3 we will show two applications of second-order three-point correlation: firstly, the physical interpretation on non-equilibrium turbulence; secondly, the inconsistency between numerical interpolation and three-point scaling law in calculations, and a preliminary model to correct this problem.

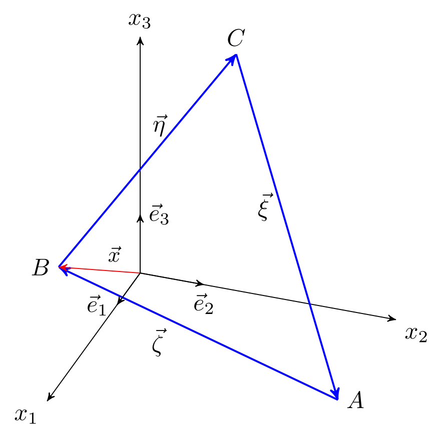

Fig. 1. Sketch of the general second-order three-point correlation on velocity increments. A, B, C are three fluid particles in HIT, locating at $\vec{x}-\vec{\zeta}$, $\vec{x}$, and $\vec{x}+\vec{\eta}$ in a Cartesian coordinate system (x_1, x_2, x_3) with unit base vectors $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ respectively.

2. Theoretical results

In this section we will introduce a new definition of second-order three-point correlation on longitudinal velocity increments in HIT. In Sec. 2.1 we will derive a general tensor formula of second-order three-point correlation on velocity increments related to traditional two-point structure functions at arbitrary three points; in Sec. 2.2 the second-order three-point correlation on longitudinal velocity will be studied and its mathematical expression will be presented.

2.1. General expressions of second-order three-point correlation on velocity increments

Fig. 1 shows a general sketch of the second-order three-point correlation on velocity increments in a Cartesian coordinate (x_1, x_2, x_3) with its unit base vectors $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$. The velocity increments are defined as

$$\Delta u_i(\vec{r}) = \vec{e}_i \cdot (\vec{u}(\vec{x}+\vec{r}) - \vec{u}(\vec{x})), \tag{1}$$

with $\vec{e}_i$ a unit base vector in i-th direction and $\vec{u}(\vec{x})$ the fluid velocity at the position of $\vec{x}$.

Similar to the definition of velocity correlation tensor, the general expression of second-order three-point correlation tensor on velocity increments is written as

$$I_{ij}(\vec{\zeta}, \vec{\eta}) := \langle \Delta u_i(\vec{\zeta}) \Delta u_j(\vec{\eta}) \rangle, \tag{2}$$

with $\langle\cdot\rangle$ the ensemble average, and $\Delta u_i(\vec{\zeta}) = \vec{e}_i \cdot (\vec{u}(\vec{x}) - \vec{u}(\vec{x}-\vec{\zeta}))$ and $\Delta u_j(\vec{\eta}) = \vec{e}_j \cdot (\vec{u}(\vec{x}+\vec{\eta}) - \vec{u}(\vec{x}))$ denoting two different velocity increments with the displacement differences $\vec{\zeta}$ and $\vec{\eta}$, which constitute a triangle in the fluid space.