Physics Letters A 382 (2018) 1563–1570

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Information and complexity measures in the interface of a metal and a superconductor

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ARTICLE INFO

Article history:
Received 21 December 2017
Received in revised form 19 March 2018
Accepted 9 April 2018
Available online 10 April 2018
Communicated by M.G.A. Paris

Keywords:
Information theory
Fisher information
Shannon entropy
Tsallis entropy
Statistical complexity
Superconductivity

ABSTRACT

Fisher information, Shannon information entropy and Statistical Complexity are calculated for the interface of a normal metal and a superconductor, as a function of the temperature for several materials. The order parameter $\Psi(\mathbf{r})$ derived from the Ginzburg–Landau theory is used as an input together with experimental values of critical transition temperature T_c and the superconducting coherence length ξ_0. Analytical expressions are obtained for information and complexity measures. Thus T_c is directly related in a simple way with disorder and complexity. An analytical relation is found of the Fisher Information with the energy profile of superconductivity i.e. the ratio of surface free energy and the bulk free energy. We verify that a simple relation holds between Shannon and Fisher information i.e. a decomposition of a global information quantity (Shannon) in terms of two local ones (Fisher information), previously derived and verified for atoms and molecules by Liu et al. Finally, we find analytical expressions for generalized information measures like the Tsallis entropy and Fisher information. We conclude that the proper value of the non-extensivity parameter $q \simeq 1$, in agreement with previous work using a different model, where $q \simeq 1.005$.

1. Introduction

Fisher information [1] has two basic roles to play in theory [2]. First, it is a measure of the ability to estimate a parameter; this makes it a cornerstone of the statistical field of study called parameter estimation. Second, it is a measure of the state of disorder of a system or phenomenon, an important aspect of physical theory [2]. Fisher's information measure (FIM) is defined for the simplest case of one-dimensional probability distribution $P(x)$ as a functional of $P(x)$ i.e.

$$\mathcal{I}_F \equiv \int P(x)\left(\frac{d\ln P(x)}{dx}\right)^2 \mathrm{d}x = \int \frac{1}{P(x)}\left(\frac{dP(x)}{dx}\right)^2 \mathrm{d}x. \quad (1)$$

$\mathcal{I}_F$ can also be written in terms of the so called Fisher information density $i(x) = \frac{1}{P(x)}\left(\frac{dP(x)}{dx}\right)^2$ i.e. $\mathcal{I}_F = \int i(x)dx$. The FIM, according to its definition, is an accounter of the sharpness of the probability density. A sharp and strongly localized probability density gives rise to a larger value of Fisher information. Its appealing features differ appreciably from other information measures because of its *local* character, in contrast with the *global* nature of several other functionals, such as the Shannon [3], Tsallis [4,5] and Renyi [6] entropies. The local character of Fisher information shows an enhanced sensitivity to strong changes, even over a very small-sized region in the domain of definition, because it is as a functional of the distribution gradient.

Very interesting applications of Fisher information have been made in quantum systems [7–17] including atoms, molecules, nuclei, or in mathematical physics in general [18–39]. However, in spite of extensive applications of information theory in quantum many body systems, most of the them focus on the behavior of systems in a single phase. Actually, there are a limited number of information-theoretical applications to systems that undergo a phase transition (see for example [40–46]). The main motivation of the present work is to extend the application of FIM in order to include systems in a phase transition by employing the phenomenological theory of Ginzburg–Landau. More precisely, we consider the order parameter $\Psi(\mathbf{r})$ as the basic ingredient of the FIM functional. In particular, we *replace* the probability distribution $P(x)$ with the inhomogeneous distribution of the superconducting phase defined in a proper way. In this model, the Fisher information measure is introduced in a convenient way, directly related with the characteristics of a phase transition. We focus on the case of the interface between a normal metal and a superconductor. We find the dependence of the FIM on specific properties of the inter-

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https://doi.org/10.1016/j.physleta.2018.04.022

face of superconductors including the superconducting coherence length as well as the critical transition temperature.

Next, we calculate the Shannon information entropy [3] and the LMC (statistical) complexity [47,48]. The dependence of these measures on the temperature is examined and interesting comments are made. The question of generalized (non-extensive) information measures is also addressed. An analytical relationship between Shannon and Fisher information, previously proved and shown to hold for atoms and molecules, is demonstrated in the present case of the superconducting interface as well.

This letter is organized as follows. In section 2 we review briefly the Ginzburg–Landau theory for inhomogeneous systems. In Section 3 we present the Fisher information measure using a probability distribution related with the order parameter. In section 4 we calculate and discuss some additional information and complexity measures including generalized ones. In section 5 we demonstrate Liu's identity, while in section 6 we discuss the possibility of alternative probability distributions. Finally, section 7 contains our concluding remarks.

2. Ginzburg–Landau theory for inhomogeneous systems

The Ginzburg–Landau theory is a theory of second-order phase transition, where one introduces an *order parameter* $\Psi(\mathbf{r})$ which is zero above the transition temperature T_c, but takes a finite value for $T < T_c$ and uses the symmetry of the relevant Hamiltonian to restrict the form of the free energy as a functional of $\Psi(\mathbf{r})$ [49]. Following the discussion by Leggett [49], we assume that $F[\Psi(\mathbf{r}), T]$ is the space integral of a free energy density $\mathcal{F}$ which is a function only of $\Psi(\mathbf{r})$ and its space derivatives and also their complex conjugates. The form of the free energy functional is [49]

$$F_s(T) = \int \mathcal{F}_s[\Psi(\mathbf{r}), T]\mathrm{d}\mathbf{r}, \tag{2}$$

$$\mathcal{F}_s[\Psi(\mathbf{r}), T] \equiv \mathcal{F}_n(T) + \alpha(T)|\Psi(\mathbf{r})|^2 + \frac{1}{2}\beta(T)|\Psi(\mathbf{r})|^4 + \gamma(T)|\nabla\Psi(\mathbf{r})|^2, \tag{3}$$

where $\mathcal{F}_s$ is the *superconducting* free energy density and $\mathcal{F}_n$ is the *normal-state* free energy density. In eq. (3) the normalization of the order parameter $\Psi(\mathbf{r})$ is arbitrary. One demands that $\Psi(\mathbf{r})$ should be zero above T_c and take a uniform non-zero value for $T < T_c$ leading to the following equalities for the coefficients $\alpha(T)$, $\beta(T)$ and $\gamma(T)$ [49]

$$\begin{aligned} \alpha(T) &\cong \alpha_0(T - T_c), \\ \beta(T) &\cong \beta(T_c) \equiv \beta, \\ \gamma(T) &\cong \gamma(T_c) \equiv \gamma. \end{aligned} \tag{4}$$

To obtain the total free energy we must integrate this over the system [49,51]

$$F_s(T) = F_n(T) + \int \left(\alpha(T)|\Psi(\mathbf{r})|^2 + \frac{1}{2}\beta|\Psi(\mathbf{r})|^4 + \gamma|\nabla\Psi(\mathbf{r})|^2\right)\mathrm{d}\mathbf{r}. \tag{5}$$

According to Eq. (5) the free energy is a functional of the scalar functions $\Psi(\mathbf{r})$ and $\Psi^*(\mathbf{r})$. In order to find the order parameter $\Psi(\mathbf{r})$ we must minimize the total free energy of the system. The condition for the minimum free energy is found by performing a functional differentiation with respect to the above functions that is to solve the following two equations

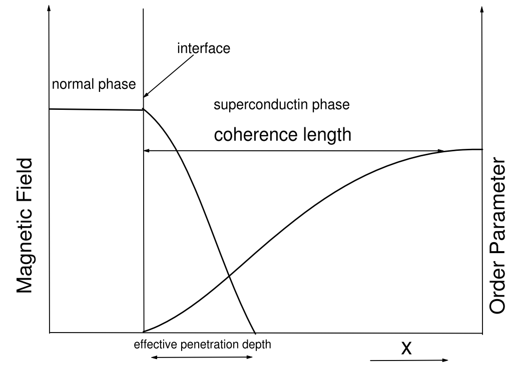

Fig. 1. A schematic picture for the spatial variation of the order parameter $\Psi(x)$ at the interface between a normal and superconducting metal in the presence of a magnetic field. The effective penetration depth λ_{eff} is also displayed. For more details see text and Ref. [50].

$$\frac{\delta F_s[\Psi(\mathbf{r}), T]}{\delta\Psi(\mathbf{r})} = 0, \quad \frac{\delta F_s[\Psi(\mathbf{r}), T]}{\delta\Psi^*(\mathbf{r})} = 0. \tag{6}$$

The above conditions can be satisfied only when $\Psi(\mathbf{r})$ obeys

$$-\gamma\nabla^2\Psi(\mathbf{r}) + \left(\alpha + \beta|\Psi(\mathbf{r})|^2\right)\Psi(\mathbf{r}) = 0. \tag{7}$$

Equation (7) has several applications including the properties of the surfaces and interface of superconductors. Following Refs. [50,51] we consider a simple model for the interface between a normal metal and a superconductor. The interface lies in the yz plane separating the normal metal in the $x < 0$ region from the superconductor in the $x > 0$ region. On the normal metal side of the interface the superconducting order parameter $\Psi(\mathbf{r})$ must be zero. Now, assuming that $\Psi(\mathbf{r})$ must be continuous, then the following one dimensional nonlinear type Schrödinger equation must be solved,

$$-\gamma\frac{\mathrm{d}^2\Psi(x)}{\mathrm{d}x^2} + \alpha(T)\Psi(x) + \beta\Psi^3(x) = 0 \tag{8}$$

in the region $x > 0$ with the boundary condition $\Psi(0) = 0$, eq. (8) can be solved analytically with the result [50]

$$\Psi(x) = \Psi_0 \tanh\left(\frac{x}{\sqrt{2}\xi(T)}\right). \tag{9}$$

Fig. 1 sketches the spatial variation of the order parameter $\Psi(x)$ at the interface between a normal and superconducting metal in the more general case of the presence of a magnetic field. The effective penetration depth λ_{eff} of the magnetic field is also displayed (for more details see Ref. [50]). In Eq. (9) Ψ_0 is the value of the order parameter in the bulk far from the surface and the parameter $\xi(T)$ is defined as

$$\xi(T) = \left(\frac{\gamma}{a(T)}\right)^{1/2} = \left(\frac{\gamma}{\alpha_0}\right)^{1/2}\frac{1}{\sqrt{T_c - T}}. \tag{10}$$

Considering that $\xi(0) \equiv \xi_0$ is the value of ξ for $T = 0$ the above relation is rewritten as

$$\xi(T) = \xi_0\frac{1}{\sqrt{1 - \frac{T}{T_c}}}, \quad \xi_0 = \left(\frac{\gamma}{a_0 T_c}\right)^{1/2}. \tag{11}$$

The quantity ξ has dimensions of length and is known as the Ginzburg–Landau coherence length or healing length. The physical significance of this length, in the condensed phase, is that it is a measure of the minimum distance over which one can "bend" the order parameter either in magnitude or in phase, before the bending energy becomes comparable to the condensation energy