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Hybrid lattice Boltzmann finite difference simulation of mixed convection flows in a lid-driven square cavity

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ABSTRACT

Mixed convection heat transfer in two-dimensional lid-driven rectangular cavity filled with air ($Pr = 0.71$) is studied numerically. A hybrid scheme with multiple relaxation time lattice Boltzmann method (MRT-LBM) is used to obtain the velocity field while the temperature field is deduced from energy balance equation by using the finite difference method (FDM). The main objective of this work is to investigate the model effectiveness for mixed convection flow simulation. Results are presented in terms of streamlines, isotherms and Nusselt numbers. Excellent agreement is obtained between our results and previous works. The different comparisons demonstrate the robustness and the accuracy of our proposed approach.

1. Introduction

Nowadays, the *Lattice Boltzmann Method* (LBM) is considered as an alternative numerical method which has attracted much attention as a technique in fluid engineering [1]. This method based on a mesoscopic study of the macroscopic problem incorporates the basic conservation laws of the hydrodynamic variables such as density and velocity. This approach is initially developed from its predecessor, the *Lattice Gas Automata* (LGA). The LBM has rapidly evolved into a self-standing research subject. Thereafter, it has to be an efficient tool for simulating problems of fluid mechanics and transport phenomena [2–7]. A literature survey shows that this method is also used for applications involving interfacial dynamics and complex boundaries such as multiphase flows [8–10], compressible flows [11,12] and porous media [13]. Moreover, LBM is well-suited for high-performance implementations on massively parallel processors such as, for example, graphics processing units (GPUs) [14].

Concerning the term of collision in the lattice Boltzmann equation, two types of collision operator are considered. One of the simplest and most widely used models proposed by Bhatnagar, Gross and Krook [15], called BGK model, based on a single relaxation time (SRT). It achieved considerable success due to its easy implementation and the ability to take into account complex geometries [16–18]. Despite the great advantages, this model, with single relaxation time, reveals deficiencies due to the numerical instabilities [19] and consequent difficulties to reach high Reynolds number flows. This deficiency can be easily treated by using the second type of collision operator called Multiple Relaxation Time (MRT) operator [20–22]. The MRT model presents numerous advantages compared to the BGK model. It leads to a stable solution for flows with higher Reynolds numbers.

The main limitation of using LBM in engineering applications is the lack of satisfactory model for the thermal fluid flows problems. To remedy this problem, several approaches have been proposed which can be grouped into three categories: multispeed approach, double population approach and hybrid approach.

The multispeed model [23,24] consists in extending the distribution function in order to obtain the macroscopic temperature. The authors of previous works [23–25] concluded that this model is not advantageous because it requires more computational resources and is less stable than the other approaches described below [25].

The double population approach, called passive scalar approach, has been proposed to use two independent distribution functions [26,27]. One for the velocity field and the other for the temperature field. In this approach the temperature is considered as a passive scalar transported by the speed without changing the velocity field. This model assumes that the viscous dissipation and compression work can be neglected for incompressible fluids and the

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evolution of the temperature is given by the advection–diffusion equation. However, this approach is considered ineffective [28] because it is not necessary to add a distribution function to simulate a passive scalar.

Concerning the hybrid model, used in this article, and according to Lallemand and Luo [28], the instability of previous models inherent the LB method and is due to a coupling between the modes of collision operator. Authors show that the fault cannot be eliminated by increasing the number of speeds. Therefore, they argue that the best alternative to build athermal model is to use a hybrid method in which the flow is determined by the LB method and the energy equation is solved by another method.

For this reason, in our work the LBM-MRT model is used for velocity field, on the one hand, and finite differences for temperature field, on the other hand. First, the method is validated for the classical MRT lid-driven cavity, yielding satisfactory agreement with data from the literature. Thereafter, the model has been used to simulate the 2D mixed convective flow in a lid-driven square cavity.

This paper is organized as follows. Section 2 presents the hybrid multiple relaxation time Lattice Boltzmann Method with (D_2Q_9) lattice model to simulate the fluid flow. In Section 3 we present the finite difference method (FDM) to solve the energy equation. The problem description and boundary conditions are presented in Section 4. Results and discussions are presented in Sections 5.1 and 5.2. In Section 5.1 the MR-lid-driven cavity is presented. Section 5.2 deals with the coupling between the hybrid lattice Boltzmann method with the finite difference method for simulating mixed convection. Finally, we draw some conclusions.

2. Multiple relaxation time lattice Boltzmann method (MRT-LBM)

Within the LBM approach, fluid is described by a particle distribution function which evolves in discrete space and time (a D_dQ_q lattice, d dimensions and q velocities) following two steps: propagation and collision. Hence, the lattice Boltzmann equation is expressed as:

$$f_i(\vec{x}+\vec{e}_i, t+1) - f_i(\vec{x}, t) = \Omega_i \tag{1}$$

where f_i is the probability of finding a particle at lattice node $\vec{x}$, at the time t, moving with velocity $\vec{e}_i$ $(i = 0, \dots, q-1)$ and Ω_i is the collision operator. Note that the time step is made unit by convention.

It is more convenient to perform the collision process in the moment space, a square matrix can be used to represent the transformation. The discrete distribution function f_i could be expressed in terms of moments m_i, by $|m\rangle = \mathbf{M}|f\rangle$. $\mathbf{M}$ is a matrix constricted from velocity [19]. Hence, collision is expressed as moment relaxation:

$$m_i^* = m_i - s_i\left(m_i - m_i^{eq}\right) \tag{2}$$

where m_i^{eq} is the equilibrium moment, m_i^* is the post collision moment and s_i is the relaxation matrix rate.

Physically moments are given by:

$$|m\rangle = (\rho \quad e \quad \epsilon \quad j_x \quad q_x \quad j_y \quad q_y \quad p_{xx} \quad p_{xy})^\top$$

In the above, ρ is the density, e is the energy mode, ϵ is defined as the kinetic energy, j_x and j_y the x and y components of momentum (mass flux), q_x and q_y correspond to the x and y components of the energy flux. In addition, p_{xx} and p_{xy} correspond to the diagonal and off-diagonal components of the viscous stress tensor, and $\top$ denotes the transpose operator.

The macroscopic variables such as density ρ, velocity $\vec{u}$ are calculated as the moments of the distribution functions:

$$\rho = \sum_{i=0}^{q-1} f_i \quad \text{and} \quad \rho\vec{u} = \sum_{i=0}^{q-1} f_i\vec{e}_i \tag{3}$$

The nine velocity square lattice Boltzmann model (D_2Q_9), as shown in Fig. 2, has been used in our work due to its widely and successfully simulation of the two-dimensional thermal flows.

For the (D_2Q_9) lattices, the particle speeds $\vec{e}_i$ are defined as:

$$\begin{cases} \vec{e}_i = \vec{0} & i = 0 \\ \vec{e}_i = \left(\cos\left[(i-1)\frac{\pi}{2}\right], \sin\left[(i-1)\frac{\pi}{2}\right]\right)c & i = 1,2,3,4 \\ \vec{e}_i = \left(\cos\left[(2i-9)\frac{\pi}{4}\right], \sin\left[(2i-9)\frac{\pi}{4}\right]\right)c\sqrt{2} & i = 5,6,7,8 \end{cases} \tag{4}$$

where $c = \frac{\Delta x}{\Delta t}$ is the lattice speed, Δx and Δt are the lattice width and time step, respectively. It is chosen that $\Delta x = \Delta t$, thus $c = 1$.

In D_2Q_9 lattice, the matrix $\mathbf{M}$ which constructed from velocities is defined as [19]:

$$\mathbf{M} \equiv \begin{pmatrix} \rho \\ e \\ \epsilon \\ j_x \\ q_x \\ j_y \\ q_y \\ p_{xx} \\ p_{xy} \end{pmatrix} \equiv \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\ 4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & -2 & 0 & 2 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \end{pmatrix} \tag{5}$$

where the equilibrium value of moments can be also derived from the following equations:

$$\begin{cases} \rho^{eq} = \rho \\ e^{eq} = -2\rho + 3\left(u^2+v^2\right) \\ \epsilon^{eq} = \rho - 3\left(u^2+v^2\right) \\ j_x^{eq} = \rho u \\ j_y^{eq} = \rho v \end{cases} \quad \text{and} \quad \begin{cases} q_x^{eq} = -u \\ q_y^{eq} = -v \\ p_{xx}^{eq} = u^2 - v^2 \\ p_{xy}^{eq} = u.v \end{cases} \tag{6}$$

The equilibrium density distribution function which depends on the local velocity and density is given by:

$$f_i^{eq} = w_i\rho\left[1 + \frac{3\vec{e}_i\cdot\vec{u}}{c^2} + \frac{9(\vec{e}_i\cdot\vec{u})^2}{2c^4} - \frac{3\vec{u}\cdot\vec{u}}{2c^2}\right] \quad i = 0 \to 8 \tag{7}$$

where w_i is the weighting factor defined as:

$$\begin{cases} w_i = \frac{4}{9} & i = 0 \\ w_i = \frac{1}{9} & i = 1,2,3,4 \\ w_i = \frac{1}{36} & i = 5,6,7,8 \end{cases} \tag{8}$$

The relaxation rates can be expressed in a matrix form as:

$$S = diag(S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8) \tag{9}$$

In the present work we assume $S_0 = S_3 = S_5 = 0$ for mass and momentum conservation before and after collision [19]. We also consider $S_7 = S_8 = \frac{1}{\tau}$ due to fact that the viscosity formulation is the same as well as SRT model [19]. In the present simulation, $S_1 = 1.64$, $S_2 = 1.2$ and $S_4 = S_6 = 8 \times \frac{(2-S_7)}{(8-S_7)}$.

In the LBM the kinematic viscosity ν is related to the relaxation time by the relation:

$$\nu = (\tau - 0.5)c_s^2\Delta t \tag{10}$$