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Analytical solution of convective heat transfer of a quiescent fluid over a nonlinearly stretching surface using Homotopy Analysis Method

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ABSTRACT

In this article, an analytical solution of the boundary layer fluid flow and heat transfer of a quiescent viscous fluid over a non-linearly stretching surface is presented. The thermal radiation effects are included in the energy governing equation. Surface velocity and temperature conditions are assumed to be of the power-law form with an exponent of 1/3 for velocity and arbitrary exponent *m* for surface temperature or heat flux conditions. The system of nonlinear differential equations is solved by Homotopy Analysis Method (HAM) for two cases of Prescribed Surface Temperature (PST) and Prescribed Heat Flux (PHF). The results of this method appear in the form of series expansions, the convergence of which is analyzed carefully. Graphical results are finally presented in order to investigate the influence of Prandtl number (*Pr*) and thermal radiation on heat transfer phenomena.

Introduction

A large number of engineering and scientific problems can be appropriately formulated in the form of a system of non-linear partial differential equations subject to specific boundary and initial conditions. However, a general methodology in order to analytically tackle with such systems of equations still lacks. The most common choice for complicated problems seems to be numerical computations, in which values of dependent variables are accounted for at only a set of discrete points (commonly referred to as computational nodes). Nevertheless, the analytical solutions are highly favored over numerical results especially because of their ability to continuously describe the variable fields and the lack of a number of errors that are typically associated with the numerical solutions.

There are many approximate analytical methods, which have been developed and extensively used for the case of ordinary non-linear differential equations. In the case of partial differential equations if a similarity variable exists, the system of differential equations is possible to be transformed to a system of ordinary differential equations. Then the analytical methods that are applicable to ODEs can be utilized in order to resolve the resulting system of ODEs.

There are many well-known method to solve equations such as DQ, GDQ, DTM and etc [1–4]. One of the most recently applied approximate analytical methods, for solving non-linear ordinary differential equations, is Homotopy Analysis Method (HAM), which was developed by Liao in 1992 [5]. It is the general form of HPM [6], ADM [7] and δ-expansion methods, which overcomes the restriction of requiring a small parameter in the foregoing methods [8]. In order to make sure about its validity and accuracy it was widely applied by many researchers for a variety of problems such as viscous boundary layer flow due to a moving sheet[9], viscos flow on flat plate [10], Blasius viscos flow [11], non-Newtonian fluids over a sheet [12], hydro magnetic nano-fluids [13], vibration of beams [14], boundary layer flows and heat transfer subject to non-linear boundary conditions [15–17].

Ajam et al. [18] applied Buongiorno's Model to study a surface stretching with convective conditions in a magnetohydrodynamic (MHD) nano-fluid. They showed by increasing Lewis number, the species boundary layer thins and the concentration profiles become steeper. Abbasbandy used Homotopy Analysis Method in many heat transfer problems such as heat radiation equations [19]. Marinca and Herişanu [20] investigated nonlinear equations arising in heat transfer bu OHAM. Sheikholeslami et al. [21] studied nanofluid flow over a stretching plate in existence of magnetic field by using Buongiorno Model. They showed by increasing porosity and melting parameters the Nusselt number reduced.

There are many industrial and chemical processes where a surface that is being stretched or drawn needs to be cooled before taking any further packaging or processing action. The cooling effect is typically

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achieved by means of a fluid surrounding the surface, e.g. paper production, manufacturing electronic chips, iron forming, and many others. As of yet several engineering problems concerning stretching surfaces have been mathematically modeled under various conditions and using both Newtonian and non-Newtonian working fluids [22–27].

To name a few, heat transfer from a moving hot surface and nonlinear stretching effects on fluid flow were studied by Chen [28], and Vajravelu [29], respectively. Power law and exponentially stretching surface have been investigated by Ali [30] and Elbashbeshy [31], respectively. Cortell [32]conducted a numerical solution of the flow and heat transfer over a nonlinearly stretching sheet, using Runge-Kutta integration method.

In the present study, the system of non-linear equations of the fluid flow and heat transfer of a viscous quiescent fluid over a non-linearly stretching surface are analytically solved by HAM. The thermal radiation is also taken into account and the problem is solved subject to two sets of thermal boundary conditions, i.e., Prescribed Surface Temperature (PST) and Prescribed Heat Flux (PHF). The results showed that by increasing surface temperature parameter, dimensionless temperature in both PST and PHF cases decrease. Also dimensionless stream function, velocity and dimensionless temperature in both cases of PST and PHF are represented graphically.

Flow and energy analysis and mathematical formulation

Governing equations of fluid flow

As shown in Fig. 1, let us consider a Newtonian incompressible flow over a flat stretching surface which corresponds to $y = 0$ in the Cartesian coordinates system depicted. The surface is being stretched non-uniformly in x direction. The 2-dimensional constant-property boundary layer equations are expressed as Eqs. (1) and (2), which are the statements of conservation of mass and linear momentum in x direction, respectively. These governing equations are subject to the assumption of slender boundary layer which holds true for the case of high Re numbers ($Re = \bar{u}_w L/\nu$). The main consequences of slenderness assumption are $v \ll u$ and $\partial^2 u/\partial x^2 \ll \partial^2 u/\partial y^2$.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}. \tag{2}$$

Boundary conditions for the present problem as set forth by Cortell [32], are

$$u_w(x) = \frac{\nu}{L^{4/3}}x^{1/3}, \quad v = 0 \ at \ y = 0, \tag{3}$$

$$u \to 0 \quad as \quad y \to \infty. \tag{4}$$

In which L is the characteristic length of the surface taken as the stream-wise surface extent. As noticed the wall stretching velocity appears as a nonlinear boundary condition, Eq. (3). Similarity variables are defined as follows [32]:

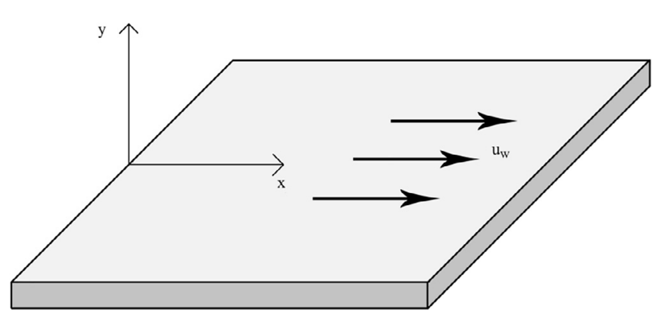

Fig. 1. The stretching surface, with surface velocity u_w.

$$\eta = y\frac{x^{-1/3}}{L^{2/3}}, \quad u = \frac{\nu}{L^{4/3}}x^{1/3}f'(\eta), \quad v = -\frac{\nu}{L^{2/3}}x^{-1/3}\frac{2f-\eta f'}{3}. \tag{5}$$

Eq. (2) is reduced to the following nonlinear differential equation [32]

$$3f''' + 2ff'' - (f')^2 = 0, \tag{6}$$

where f is the dimensionless stream function and the primes denote differentiation with respect to the similarity variable, η. The Eq. (6) is subject to boundary conditions given below:

$$f = 0, \quad f' = 1 \ at \ \eta = 0, \tag{7}$$

$$f \to 0 \quad as \quad \eta \to \infty. \tag{8}$$

Governing equation of heat transfer

The boundary layer including thermal radiation is given by Eq. (9) as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}. \tag{9}$$

In which T is temperature, α the thermal diffusivity, ρ the fluid density, C_p is the fluid specific heat at constant pressure, and q_r is the radiative heat flux. Using Rosseland approximation [33] the radiative heat flux is simply expressed as follows:

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}, \tag{10}$$

where σ is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. By expanding T^4 using a Taylor series about T_∞ and neglecting higher-order terms Eq. (10) is simplified and then substituted in Eq. (9). Thus, the energy equation takes the following form of [32]:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha + \frac{16PrT_\infty^3}{3\rho C_p k^*}\right)\frac{\partial^2 T}{\partial y^2}. \tag{11}$$

Prescribed surface temperature (PST) case

The nonlinear boundary condition for surface temperature is considered as follows, with $m \neq 0$ and $m \neq 1$:

$$T_w = T_\infty + A\left(\frac{x}{L}\right)^m \ at \ y = 0, \quad T \to T_\infty \ as \ y \to \infty, \tag{12}$$

where A is a constant, and T_∞ is the free stream fluid temperature. For convenience the dimensionless temperature θ is defines as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{13}$$

Using similarity variable the Eqs. (11) and (12) are reduced to [32]:

$$\theta'' + \frac{2k_0}{3}Prf\theta' - Prk_0 mf'\theta = 0, \tag{14}$$

$$\theta = 1 \quad at \quad \eta = 0; \theta \to 0 \quad as \quad \eta \to \infty. \tag{15}$$

In which $Pr(=\nu/\alpha)$ is the fluid Prandtl number, $N_R = k^*k/4\sigma$ the radiation parameter [32], and we also have $k_0 = 3N_R/(3N_R + 4)$.

Prescribed heat flux (PHF) case

In this case, dimensionless temperature is defined as [32]

$$g(\eta) = \frac{T - T_\infty}{\left(\frac{D}{k}\right)x^{m+1/3}L^{2/3-m}}, \tag{16}$$

and the corresponding boundary conditions are