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Stagnation point flow of viscoelastic nanomaterial over a stretched surface

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ABSTRACT

Present communication aims to discuss magnetohydrodynamic (MHD) stagnation point flow of Jeffrey nanofluid by a stretching cylinder. Modeling is based upon Brownian motion, thermophoresis, thermal radiation and heat generation. Problem is attempted by using (HAM). Residual errors for h-curves are plotted. Convergent solutions for velocity, temperature and concentration are obtained. Skin friction coefficient, local Nusselt number and Sherwood number are studied. It is examined that velocity field decays in the presence of higher estimation of magnetic variable. Furthermore temperature and concentration fields are enhanced for larger magnetic variable.

1. Introduction

The thermal nanofluids for heat transfer applications represent a class of its own difference from conventional colloids for other applications. Heat transfer has many utilization in industries with the purpose of both increasing and decreasing temperature. Conventional fluids such as ethylene glycol, water, oil, etc. are conventional materials which do not have higher thermal conductivity. Nano-sized particles enhance the rate of heat transfer when used in a base fluid. These particles are useful in a wide range of applications in industry, transportation, microelectronics and thermal generation. Choi and Eastman [1] investigated an enhancement in thermal conductivity of conventional fluids via addition of nanoparticles. Nanoparticles are available in various shapes, e.g. rod-like, tabular or spherical. Turkyilmazoglu et al. [2] discussed behavior of heat and mass transfer of nanoparticles during flow by vertical flat plate with effects of thermal radiation. Few recent studies about thermophoresis and Brownian motion are mentioned through Refs. [3–10].

Recent researchers also discussed about different characteristic of non-Newtonian models. Rate type fluids are the sub-class of non-Newtonian fluids. These materials describe contributions of relaxation and retardation times. Jeffrey fluid model [11–14] is suggested to describe the same effect. Hayat et al. [15] described stagnation point flow of MHD second-grade fluid by a shrinking cylinder. Sheikholeslami et al. [16] explored MHD flow with effects of heat and mass transfer over a shrinking surface. Further radiation effect cannot be ignored for higher temperature. This effect in boundary layer is prominent in engineering, industries, space technology and in nuclear reactors etc. Pal [17] investigated the behavior of Hall current and thermal radiation on MHD flow bounded by an unsteady shrinking surface. Bhattacharyya et al. [18] discussed about the influences of radiation in micropolar fluid flow past a porous stretching surface. Mukhopadhyay [19] analyzed the radiation and slip impacts on MHD boundary layer flow. Hayat et al. [20] and Bhattacharyya [21] explored mixed convective and radiative flows of Maxwell fluid in the presence of stagnation point and thermal radiation. Das et al. [22] modeled MHD second grade flow towards convective heated extended surface. Flows by moving sheets are also discussed in the studies [23–26]. Hayat et al. [27] examined flow of Jeffrey fluid with mixed convection and double stratification effects. Viscous fluid flow by a stretching cylinder is discussed by Wang [28]. Features of Jeffrey nanofluid flow with mass flux radiation and mixed convection condition are studied in Refs. [29–31]. The references [32–45] investigated flow by a cylinder or sheet with different flow behavior like

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Cattaneo-Christov heat flux, high Reynolds number, homogeneous-heterogeneous reactions etc.

The purpose of current attempt is to report the MHD stagnation point flow of Jeffrey nanofluid bounded by a stretchable sheet. Brownian and thermophoresis effects are considered. Boundary layer concept and assumption of small magnetic Reynold number are addressed. Homotopy approach [42–45] is used to solve the governing nonlinear systems. Aspects of several significant parameters on velocity $(f'(\eta))$, temperature $(\theta(\eta))$ and nano-particles volume fraction $(\phi(\eta))$ are addressed. This study is arranged in the following structure. Next section has formulation. Section three has solutions and related analysis. Discussion and conclusions are given in sections four and five.

2. Formulation

Consider the stagnation point flow of magneto Jeffrey nanomaterial by a stretching cylinder of radius R_1. Heat generation is studied. Here x-axis is in the axial direction and r is normal to x-axis. A uniform applied magnetic field B_0 conducts the fluid. Thermophoresis and Brownian motion are considered. The problems statements are [36]:

$$\frac{\partial(rv)}{\partial r}+\frac{\partial(ru)}{\partial x}=0, \tag{1}$$

$$\left.\begin{array}{c} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=U_e\frac{dU_e}{dx}+\frac{\nu}{1+\lambda_1} \\ \left(\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}+\lambda_2\left(\begin{array}{c}\frac{v}{r}\frac{\partial^2 u}{\partial r^2}+\frac{\partial v}{\partial r}\frac{\partial^2 u}{\partial r^2}+v\frac{\partial 3u}{\partial r^3} \\ \frac{u}{r}\frac{\partial^2 u}{\partial r\partial x}+\frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial r\partial x}+v\frac{\partial^3 u}{\partial x\partial r^2}\end{array}\right)\right)-\frac{\sigma B_0^2}{\rho}(u-U_e)\end{array}\right\} \tag{2}$$

$$\left.\begin{array}{c} u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+ \\ \tau\left(D_B\frac{\partial C}{\partial r}\frac{\partial T}{\partial r}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial r}\right)^2\right)+\frac{Q_0}{\rho C_p}(T-T_\infty)-\frac{1}{\rho C_p}\frac{\partial}{r\partial r}(rq_r)\end{array}\right\}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial r}=D_B\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right), \tag{4}$$

$$\left.\begin{array}{c} u=U_w(x)=\frac{U_0 x}{l},\ v=0,\ T=T_w=T_\infty+T_0\left(\frac{x}{l}\right), \\ C=C_w=C_\infty+C_0\left(\frac{x}{l}\right)\quad \text{for } r=R_1 \\ u=U_e=\frac{U_\infty x}{l},\ T\to T_\infty,\ C\to C_\infty\quad \text{for } r\to\infty.\end{array}\right\}. \tag{5}$$

Here u and v are the velocities along x and r directions respectively. Thermal radiation contribution by Rosseland approximation leads to the expression:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial r}. \tag{6}$$

The term T^4 in (6) about T_∞ by Taylor series can be approximated to $T^4\approx 4T_\infty^3 T-3T_\infty^4$. Using

$$\left.\begin{array}{c}\psi=\sqrt{U_w\upsilon x}R_1 f(\eta),\ \eta=\sqrt{\frac{U_w}{\upsilon x}}\left(\frac{r^2-R_1^2}{2R_1}\right),\ u=U_w f'(\eta),\ \mathrm{v}=-\sqrt{\frac{\nu U_0}{l}}\frac{R_1}{r}f(\eta), \\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_\infty}.\end{array}\right\}, \tag{7}$$

Eq. (1) is trivially satisfied and Eqs. (2)–(4) along with boundary conditions can be put into the forms:

$$\left.\begin{array}{c}(1+2\gamma\eta)f'''+(1+\lambda_1)\left(ff''-(f')^2\right)+2\gamma f''+\gamma\beta(f'f''-3ff''')+ \\ (1+2\gamma\eta)\beta\left((f'')^2-ff^{iv}\right)-(1+\lambda_1)M^2 f'+(1+\lambda_1)\left(A^2+M^2 A\right)=0\end{array}\right\}, \tag{8}$$

$$\left.\begin{array}{c}(1+2\gamma\eta)(1+\frac{4}{3}R)\theta''+2\gamma(1+\frac{4}{3}R)\theta'+\Pr(f\theta'-f'\theta) \\ +(1+2\gamma\eta)N_b\theta'\phi'+(1+2\gamma\eta)N_t(\theta')^2+\Pr\delta\theta=0.\end{array}\right\}, \tag{9}$$

$$(1+2\gamma\eta)\phi''+2\gamma\phi'+Sc(f\phi'-f'\phi)+2\frac{N_t}{N_b}\gamma\phi'+\frac{N_t}{N_b}(1+2\gamma\eta)\theta''=0, \tag{10}$$

$$\left.\begin{array}{c}f(0)=0,\ f'(0)=1,\ \theta(0)=1,\ \phi(0)=1, \\ f'(\infty)=A,\ \theta(\infty)=0,\ \phi(\infty)=0.\end{array}\right\}, \tag{11}$$

in which prime represents derivative with respect to η. The values of $\gamma,\ A,\ \beta,\ M,\ R,\ N_t,\ N_b,\ Sc,\ \Pr,\ \tau,\ \delta$ are:

$$\left.\begin{array}{l}\gamma=\left(\frac{\upsilon l}{U_0 R_1^2}\right)^{\frac{1}{2}},\ A=\frac{U_\infty}{U_0},\ \beta=\frac{\lambda_2 U_0}{l},\ M=\sqrt{\frac{\sigma l}{\rho U_0}}B_0, \\ R=\frac{4\sigma^* T_\infty^3}{3kk^*},\ N_t=\frac{(\rho C)_p}{(\rho C)_f}\frac{D_T(T_w-T_m)}{T_\infty\nu},\ N_b=\frac{(\rho C)_p}{(\rho C)_f}\frac{D_B(C_w-C_\infty)}{\nu}, \\ Sc=\frac{\upsilon}{D_B},\ \Pr=\frac{\mu C_p}{k},\ \tau=\frac{(\rho C)_p}{(\rho C)_f},\ \delta=\frac{lQ_0}{\rho C_p U_0}.\end{array}\right\}. \tag{12}$$

Here γ is defined as the curvature parameter, A the ratio of velocities, β the Deborah number, M the Hartman number, R the radiation parameter, N_t and N_b the thermophoresis parameter and the Brownian diffusion parameter respectively, Sc the Schmidt number, Pr the Prandtl number, τ the ratio of liquid heat capacity to nanoparticles effective heat capacity and δ the dimensionless heat generation parameter. The local Nusselt number, skin friction coefficient and local Sherwood number are:

$$Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, \tag{13}$$

$$C_f=\frac{\tau_w}{\rho U_w^2}, \tag{14}$$

$$Sh_x=\frac{xq_m}{D_B(C_w-C_\infty)}, \tag{15}$$

with

$$q_w=-k\left(\frac{\partial T}{\partial r}\right)\Big|_{r=R_1}, \tag{16}$$

$$\tau_w=\frac{\mu}{1+\lambda_1}\left[\frac{\partial u}{\partial r}+\lambda_2\left[v\frac{\partial^2 u}{\partial r^2}+u\frac{\partial^2 u}{\partial r\partial x}\right]\right]\Big|_{r=R_1}, \tag{17}$$

$$q_m=-D_B\left(\frac{\partial T}{\partial r}\right)\Big|_{r=R_1}. \tag{18}$$

Dimensionless local Nusselt number, skin friction coefficient and Sherwood number are:

$$\frac{Nu_x}{\sqrt{Re_x}}=-\theta'(0), \tag{19}$$

$$\left.C_f\sqrt{\mathrm{Re}_x}=\frac{1}{(1+\lambda_1)}\left(f''(0)-\beta f(0)f'''(0)-\beta\gamma f(0)f''(0)+\beta f'(0)f''(0)\right)\right\}, \tag{20}$$

$$\frac{Sh_x}{\sqrt{Re_x}}=-\phi'(0), \tag{21}$$

with local Reynold number by

$$Re_x=\frac{u_w x}{\upsilon}, \tag{22}$$

where υ is the kinematic viscosity.