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Thin linearly piezoelectric junctions

Jonctions minces linéairement piézoélectriques

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ABSTRACT

Through a rigorous mathematical analysis, we present various asymptotic models for a thin piezoelectric junction between two linearly piezoelectric or elastic bodies. Depending on the relative behavior of a stiffness parameter with respect to its thickness, the joint is replaced by either a(n) (electro)mechanical constraint or a piezoelectric material surface.

RÉSUMÉ

Par une analyse mathématique rigoureuse, nous présentons divers modèles asymptotiques pour une jonction mince piézoélectrique entre deux corps linéairement élastiques ou piézoélectriques. Selon l'ordre de grandeur relatif entre un paramètre de rigidité et l'épaisseur, le joint est remplacé par une liaison (électro)mécanique ou par une surface matérielle piézoélectrique.

1. Introduction

We present various asymptotic models, indexed by $p=(p_1,p_2)\in\{1,2,3,4\}^2$, for a thin piezoelectric junction between two linearly piezoelectric ($p_2=1$) or elastic ($p_2>1$) bodies. Index p_1 is relative to the magnitude of the piezoelectric coefficients of the adhesive, characterized by a single parameter μ, with respect to that of the constant thickness 2ε of a layer containing the adhesive. More precisely, we assume that $h:=(\varepsilon,\mu)$ takes values in a countable set with a sole cluster point $\bar{h}\in\{0\}\times[0,+\infty]$ so that:

$$\begin{cases} p_1=1: \ \bar{\mu}_1:=\lim_{h\to\bar{h}}(\varepsilon\mu)\in(0,+\infty) \\ p_1=2: \ \bar{\mu}_1:=\lim_{h\to\bar{h}}(\varepsilon\mu)=0, \quad \bar{\mu}_2:=\lim_{h\to\bar{h}}(\mu/2\varepsilon)=+\infty \\ p_1=3: \ \bar{\mu}_2:=\lim_{h\to\bar{h}}(\mu/2\varepsilon)\in(0,+\infty) \\ p_1=4: \ \bar{\mu}_2:=\lim_{h\to\bar{h}}(\mu/2\varepsilon)=0. \end{cases} \tag{1}$$

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As previously said, index p_2 characterizes the status of the adherents but also that of the interfaces between adherents and adhesive:

$$\begin{cases} p_2 = 1: & \text{the two interfaces are electromechanically perfectly permeable,} \\ p_2 = 2: & \text{the two interfaces are electrically impermeable,} \\ p_2 = 3: & \text{one interface is electrically impermeable while the other is electroded,} \\ p_2 = 4: & \text{the two interfaces are electroded.} \end{cases} \tag{2}$$

The space $\mathbb{R}^3$ is assimilated with the physical Euclidean space with basis $\{e_1, e_2, e_3\}$. Let Ω be a domain, with Lipschitz-continuous boundary, whose intersection S with $\{x_3 = 0\}$ is a domain of $\mathbb{R}^2$ of positive two-dimensional Hausdorff measure $\mathcal{H}_2(S)$. Let $\Omega_\pm := \Omega \cap \{\pm x_3 > 0\}$ and ε be a small positive number, then adhesive and adherents occupy $B^\varepsilon := S \times (-\varepsilon, \varepsilon)$, $\Omega^\varepsilon_\pm := \Omega_\pm \pm \varepsilon e_3$, respectively; let $\Omega^\varepsilon = \Omega^\varepsilon_+ \cup \Omega^\varepsilon_-$, $S^\varepsilon_\pm := S \pm \varepsilon e_3$, $\mathcal{O}^\varepsilon := \Omega^\varepsilon \cup B^\varepsilon \cup_\pm S^\varepsilon_\pm$. Let $(\Gamma_{\mathrm{mD}}, \Gamma_{\mathrm{mN}})$, $(\Gamma_{\mathrm{eD}}, \Gamma_{\mathrm{eN}})$ be two partitions of $\partial\Omega$ with $\mathcal{H}_2(\Gamma_{\mathrm{mD}})$, $\mathcal{H}_2(\Gamma_{\mathrm{eD}}) > 0$ and $0 < \delta := \mathrm{dist}(\Gamma_{\mathrm{eD}}, S)$. For all Γ in $\{\Gamma_{\mathrm{mD}}, \Gamma_{\mathrm{mN}}, \Gamma_{\mathrm{eD}}, \Gamma_{\mathrm{eN}}\}$, $\Gamma_\pm$, $\Gamma^\varepsilon_\pm$, Γ^ε denotes $\Gamma \cap \{\pm x_3 > 0\}$, $\Gamma_\pm \pm \varepsilon e_3$, $\cup_\pm \Gamma^\varepsilon_\pm$, respectively; if $(\gamma_{\mathrm{D}}, \gamma_{\mathrm{N}})$ is a partition of $\gamma := \partial S$, we denote $\{\gamma_{\mathrm{D}}, \gamma_{\mathrm{N}}, \gamma\} \times (-\varepsilon, \varepsilon)$ by $\{\Gamma^\varepsilon_{\mathrm{DI}}, \Gamma^\varepsilon_{\mathrm{NI}}, \Gamma^\varepsilon_{\mathrm{lat}}\}$. The structure made of the adhesive and the two adherents, perfectly stuck together along $S^\varepsilon_\pm$, is clamped on $\Gamma^\varepsilon_{\mathrm{mD}}$, subjected to body forces of density f^ε and to surface forces of density F^ε on Γ^ε_{mN} and vanishing on $\Gamma^\varepsilon_{\mathrm{lat}}$. Moreover, a given electric potential $\varphi^h_{p_0}$ is applied on $\Gamma^\varepsilon_{\mathrm{DI}}$ and, when $p_2 = 1$, on $\Gamma^\varepsilon_{\mathrm{eD}}$, while electric charges of density d^ε appear on $\Gamma^\varepsilon_{\mathrm{NI}}$ and, when $p_2 = 1$, on $\Gamma^\varepsilon_{\mathrm{eN}}$.

If σ^h_p, u^h_p, $e(u^h_p)$, D^h_p, φ^h_p stand for the fields of stress, displacement, strain, electric displacement and electric potential, respectively, the constitutive equations of the structure, for all p_1 in $\{1, 2, 3, 4\}$, read as:

$$\begin{cases} (\sigma^h_p, D^h_p) = \mu M_{\mathrm{I}}\big(e(u^h_p), \nabla\varphi^h_p\big) & \text{in } B^\varepsilon \ \ \forall p_2 \in \{1,2,3,4\}, \\ \begin{cases} (\sigma^h_p, D^h_p) = M^\varepsilon_{\mathrm{E}}\big(e(u^h_p), \nabla\varphi^h_p\big) & \text{in } \Omega^\varepsilon \ \text{ if } p_2 = 1, \\ \sigma^h_p = a^\varepsilon_{\mathrm{E}} e(u^h_p) & \text{in } \Omega^\varepsilon \ \text{ if } p_2 > 1 \end{cases} \end{cases} \tag{3}$$

where

$$(M^\varepsilon_{\mathrm{E}}, a^\varepsilon_{\mathrm{E}})(x) = (M_{\mathrm{E}}, a_{\mathrm{E}})(x \mp \varepsilon e_3) \quad \forall x \in \Omega^\varepsilon_\pm \tag{4}$$

$$\begin{cases} (M_{\mathrm{I}}, M_{\mathrm{E}}) \in L^\infty\big(S \times \Omega; \mathrm{Lin}(\mathbb{K})\big) \text{ such that} \\ M_{\mathrm{P}} = \begin{bmatrix} a_{\mathrm{P}} & -b_{\mathrm{P}} \\ b^T_{\mathrm{P}} & c_{\mathrm{P}} \end{bmatrix}; \ \exists\kappa > 0, \quad \kappa|k|^2 \le M_{\mathrm{P}}(x)k \cdot k, \quad \forall k \in \mathbb{K} := \mathbb{S} \times \mathbb{R}^3, \ \text{a.e. } x \in \Omega, \ \forall \mathrm{P} \in \{\mathrm{I}, \mathrm{E}\} \end{cases} \tag{5}$$

and $\mathrm{Lin}(\mathbb{S}^3)$ is the space of linear operators on the space $\mathbb{S}^N$ of $N \times N$ symmetric matrices whose inner product and norm are noted $\cdot$ and $|\cdot|$ as in $\mathbb{R}^3$ (the same notations for the norm and inner product stand also for $\mathbb{K}$).

Lastly we have to add the following conditions on $S^\varepsilon_\pm$:

$$\begin{cases} p_2 = 2 & D^h_p \cdot e_3 = 0 \quad \text{on } S^\varepsilon_\pm, \\ p_2 = 3 & D^h_p \cdot e_3 = 0 \quad \text{on } S^\varepsilon_+, \quad \varphi^h_p = \varphi^h_{p_0} \text{ on } S^\varepsilon_-, \\ p_2 = 4 & \varphi^h_p = \varphi^h_{p_0} \qquad \text{on } S^\varepsilon_\pm, \end{cases} \tag{6}$$

the electric potential $\varphi^h_{p_0}$ being given on S^ε_+ or $S^\varepsilon_\pm$.

It will be convenient to use the following notations:

$$\begin{cases} \hat{k} := (\hat{e}, \hat{g}) \quad \hat{e} := e_{\alpha\beta}, \ 1 \le \alpha, \beta \le 2, \quad \hat{g} := (g_1, g_2), \quad \forall k = (e, g) \in \mathbb{K} \\ \tilde{e} \in \mathbb{S}^3; \quad \tilde{e}_{\alpha\beta} = e_{\alpha\beta}, \ 1 \le \alpha, \beta \le 2, \quad \tilde{e}_{i3} = 0, \ 1 \le i \le 3, \quad \forall e \in \mathbb{S}^3 \\ k(r) = k(v, \psi) := \big(e(v), \nabla\psi\big) \quad \forall r \in H^1(\mathcal{O}; \mathbb{R}^3 \times \mathbb{R}) \\ e(v) \in \mathcal{D}'(S; \mathbb{S}^2); \quad \big(e(v)\big)_{\alpha\beta} = \frac{1}{2}(\partial_\alpha v_\beta + \partial_\beta v_\alpha), \ 1 \le \alpha, \beta \le 2, \quad \forall v \in \mathcal{D}'(S; \mathbb{R}^3) \end{cases} \tag{7}$$

and the *same* symbol $e(\cdot)$ shall *also* stand for the symmetrized gradient in the sense of distributions of $\mathcal{D}'(\mathcal{O}; \mathbb{R}^3)$, $\mathcal{O} \in \{\mathcal{O}^\varepsilon, \Omega, \Omega \setminus S, B^\varepsilon, \Omega^\varepsilon\}$ or $\mathcal{D}'(S; \mathbb{R}^2)$. An electromechanical state with vanishing electric potential on $\Gamma^\varepsilon_{\mathrm{DI}}$ and on $\Gamma^\varepsilon_{\mathrm{eD}}$ when $p_2 = 1$ will belong to $V^\varepsilon_p := H^1_{\Gamma^\varepsilon_{\mathrm{mD}}}(\mathcal{O}^\varepsilon; \mathbb{R}^3) \times \Phi^\varepsilon_{p_2}$, with

$$\begin{cases} \Phi^\varepsilon_1 = H^1_{\Gamma^\varepsilon_{\mathrm{DI}} \cup \Gamma^\varepsilon_{\mathrm{eD}}}(\mathcal{O}^\varepsilon) \\ \Phi^\varepsilon_2 = H^1_{\Gamma^\varepsilon_{\mathrm{DI}}}(B^\varepsilon) \text{ if } \mathcal{H}_2(\Gamma^\varepsilon_{\mathrm{DI}}) > 0, \ H^1_m(B^\varepsilon) \text{ if } \mathcal{H}_2(\Gamma^\varepsilon_{\mathrm{DI}}) = 0 \\ \Phi^\varepsilon_3 = H^1_{\Gamma^\varepsilon_{\mathrm{DI}} \cup S^\varepsilon_-}(B^\varepsilon) \\ \Phi^\varepsilon_4 = H^1_{\Gamma^\varepsilon_{\mathrm{DI}} \cup_\pm S^\varepsilon_\pm}(B^\varepsilon) \end{cases} \tag{8}$$