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Review

Effects of a parametric perturbation in the Hassell mapping

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ABSTRACT

The convergence to the fixed point near at a transcritical bifurcation and the organization of the extreming curves for a parametric perturbed Hassell mapping are investigated. The evolution of the orbits towards the fixed point at the transcritical bifurcation is described using a phenomenological approach with the support of scaling hypotheses and homogeneous function hence leading to a scaling law related with three critical exponents. Near the bifurcation the decay to the fixed point is exponential with a relaxation time given by a power law. The extreming curves in the parameter space dictates the organization for the windows of periodicity, consequently demonstrating how the set of shrimp-like structures are organized.

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1. Introduction

Discrete time can be used to characterize the evolution of dynamical systems described by the so called mappings. Investigations might consider dissipative as well as non-dissipative dynamics in either 1-D or higher dimensions. The literature on iterated mappings is vast [1–6] and is always increasing. The studies on this sort of dynamical systems increased after the seminal papers of May and co-authors [7,8]. The dynamics reveals a complicated and intricate organization either as function of the control parameter or time [9,10], namely periodic orbits, bifurcations of different types including both local – transcritical, tangent, period doubling etc. – and global such as boundary crisis, merging attractor crisis, interior crisis, chaos and its different routes. The convergence to the fixed point at the bifurcation was proved to obey an homogeneous function characterized by a set of three critical exponents [11,12], while near the bifurcation the dynamics evolves to the stationary state via an exponential decay [13] whose relaxation time is characterized by a power law on the distance from the bifurcation. These four exponents define the class of universality of the bifurcation. After that the Hassell mapping [14,15] was considered to explore the evolution towards the equilibrium near the transcritical bifurcation [16].

In this paper we investigate the effects of a parametric perturbation in the dynamical properties of a Hassell mapping, with the main goal of to understand and describe the effects of the parametric perturbation in the convergence to the fixed point as well as to characterize the organization of the extreming curves in the parameter space [17,18]. The investigation to the asymptotic state will be made by using an homogeneous function [19]. The parameter space investigation shall allow us to understand the organization of the returning curves hence given the organization of the chaotic and periodic domains as a function of the control parameters [17].

This work is organized as follows: Section 2 describes the map, bifurcation diagram and the convergence to the fixed point. Section 3 is devoted to the analytical studies on the convergence to the fixed point at the transcritical bifurcation and in its neighboring. Section 4 is placed to discuss about the parameter space and the extreming curves. Finally, in Section 5 we present the discussions and conclusions.

2. The model

The model we consider is a time perturbed Hassell mapping written as

$$N_{n+1} = N_n \lambda (1 + b_n \epsilon)[1 + \alpha N_n]^{-\gamma}, \tag{1}$$

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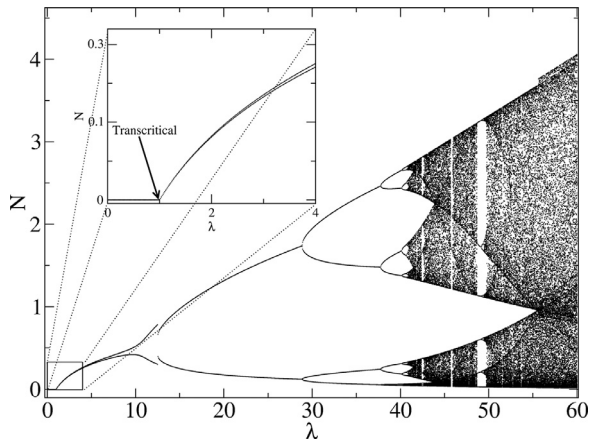

Fig. 1. Bifurcation diagram of the parametric perturbed Hassell mapping (1), using $\alpha = 1$, $\gamma = 6$, $\epsilon = 0.01$ and the initial condition $N_0 = 0.1$. The parameter λ was considered in the range $\lambda \in [0, 60]$.

where $\alpha > 0$, $\gamma > 0$ and $\lambda > 0$ are control parameters and ϵ controls the amplitude of the parametric perturbation, b_n is chosen as $b_n = (-1)^n$ or $b_n = (-1)^{n+1}$, so that for a given initial condition N_0 it can be set as $b_0 = 1$ or $b_0 = -1$. If $\epsilon = 0$ the Hassell mapping [14–16] is recovered. The time perturbation is periodic - period 2 -, it implies that, in order to find the fixed points and to study the convergence of the orbits to the stationary state, we have to calculate the map (1) in its 2th iteration, i.e., N_{n+2}. Rewriting $(n+2)$ and n in a convenient way, such that $(n+2) \to (m+1)$ and $n \to m$, we obtain

$$N_{m+1} = \lambda^2(1-\epsilon^2)N_m \times\left[(1+\alpha N_m) + \alpha\lambda(1-\epsilon)N_m(1+\alpha N_m)^{(-\gamma+1)}\right]^{-\gamma}. \quad (2)$$

Fig. 1 shows the bifurcation diagram of map (1) and we aim to investigate what happens to the behavior of the system at the fixed point $N^* = 0$ and at the transcritical bifurcation obtained from

$$\left(\frac{dN_{m+1}}{dN_m}\right)_{N_{m+1}=N^*} = 1, \quad (3)$$

yielding $\lambda_c = \frac{1}{\sqrt{(1-\epsilon^2)}}$, where the bifurcation arises (see zoom box in the Fig. 1).

To describe the convergence to the steady state we analyze the map from second iteration and looking the behavior of N approaching to the fixed point at the transcritical bifurcation. The convergence depends on the number of iterations n, on the initial condition N_0 and the parameter $\mu = \lambda_c - \lambda \cong 0$, which defines the distance from the bifurcation point. The parameter $\mu = 0$ establishes the transcritical bifurcation in the fixed point $N^* = 0$. In this way, the convergence is shown in Fig. 2 and we choose $\alpha = 1$, $\gamma = 6$, $\epsilon = 0.01$ and different initial conditions N_0 (as labeled in the figure). The procedure used can be made for another values of γ too.

A careful investigation of Fig. 2, allows us to see that different initial conditions lead to different curves with similar behaviors. They have a plateau at short n and a regime of power law decay for large enough n converging to the same fixed point. The changeover from the plateau to the decay is marked by a characteristic crossover iteration number n_x. From the behavior shown in that figure, we can suppose

1. For a short n typically $n \ll n_x$, the behavior of N_n *vs.* n is given by $N_n \propto N_0^{\alpha}$ leading to $\alpha = 1$;
2. For $n \gg n_x$, we notice $N_n \propto n^{\beta}$ where β is called a decay exponent;

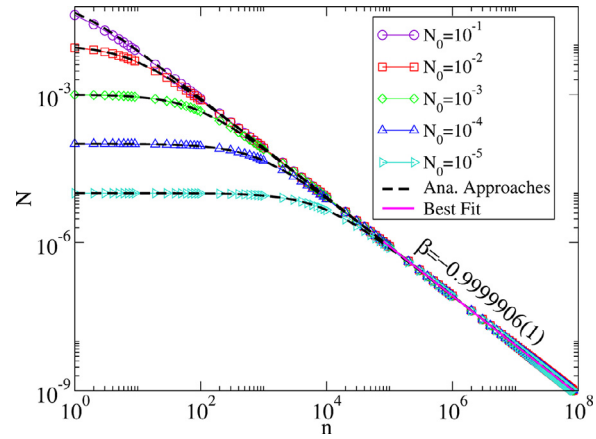

Fig. 2. Convergence to the fixed point $N^* = 0$ considering as fixed $\alpha = 1$, $\gamma = 6$, $\epsilon = 0.01$ and the different initial conditions shown in Fig. Here $\lambda_c = 1/\sqrt{(1-\epsilon^2)}$.

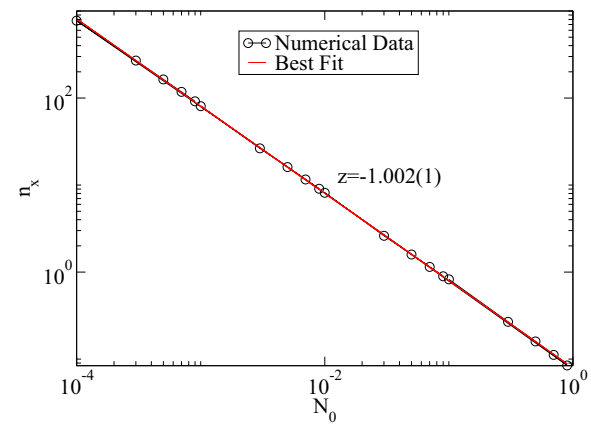

Fig. 3. Crossover iteration number n_x against the initial condition N_0, yielding through a power law fit $z = -1$.

3. Finally the crossover iteration number n_x is given by $n_x \propto N_0^z$, where z is a changeover exponent.

A power law fitting in the regime of decay in Fig. 2 gives $\beta = -0.999987(1)$. The exponent z is obtained from the plot of n_x *vs.* N_0, as shown in Fig. 3. Using $\alpha = 1$, $\gamma = 6$ and $\epsilon = 0.01$ we obtained through a power law fitting $z = -1$. The behavior shown in Fig. 2 together with the scaling hypotheses allow us to describe the behavior of N_n as homogeneous function of the variables n and N_0, when $\mu = 0$, of the type

$$N(N_0, n) = lN(l^{\tilde{a}}N_0, l^{\tilde{b}}n), \quad (4)$$

where l is a scale factor, $\tilde{a}$ and $\tilde{b}$ are characteristic exponents. Doing a similar procedure as made in Ref. [11] we obtain that

$$z = \frac{\alpha}{\beta}. \quad (5)$$

The knowledge of any two exponents allows us to find the third one by using Eq. (5). Moreover, the exponents can also be used to rescale the variables N_n and n in a convenient way, such that $N \to \frac{N}{N_0^{\alpha}}$ together with $n \to \frac{n}{N_0^z}$ to overlap all curves of N *vs.* n onto a single and hence universal curve, as shown in Fig. 4.

Let us now discuss the behavior of the convergence of the orbits to the fixed point not at the bifurcation, but rather close to it. This is the case when $\mu \neq 0$ and the decay is marked by an exponential law given by Hohenberg and Halperin [19], and Leonel et al. [20]

$$N(n, \mu) \propto e^{-n/\tau}, \quad (6)$$

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