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A comparative study of fractal dimension calculation methods for rough surface profiles

Zhiying Chen, Yong Liu*, Ping Zhou

School of Energy and Power Engineering, Beihang University, Beijing 100191, China

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ABSTRACT

Fractal dimension is the most important parameter for surface characterization. In this paper, four methods used to estimate the fractal dimensions of surface profiles and their applications in machined surfaces are studied. These methods are first evaluated using surface profiles created by Weierstrass–Mandelbrot function from the three aspects of fitting accuracy, calculation accuracy and calculation stability, and then applied to the machined rough surfaces. By comparing the results of the four methods, it is found that none of the methods is particularly prominent in all of the three aspects. However, the three point sinuosity method is found to be relatively the most suitable and reliable method among the four tested methods for extracting fractal dimensions of both generated and measured rough surface profiles.

1. Introduction

It is already known that contact mechanics of rough surfaces is important in studying and modeling physical phenomena such as friction, lubrication, wear, adhesion, fracture, etc. [1–5]. There are variety of contact mechanics models that describe the way two surfaces touch and make contact. Unfortunately, predictions from different models disagree strongly [6,7]. A recent research competition, involving more than 30 authors, was designed to evaluate these various methods and approaches [4]. However, the conclusions of this research were questioned, and even led to a debate [8,9]. Nevertheless, the contact mechanics models based on fractal theory is still considered as a promising way to solve the contact problems.

Ordinary geometric objects have integer dimensions in Euclidean geometry. Objects such as points, curves, surfaces and cubes having integer dimensions of 0, 1, 2, 3 respectively are the familiar examples. However, in recent decades, objects with fractal dimensions rather than integral dimensions have aroused widespread concerns. These kind of objects are termed as fractals. They are often disordered, irregular, and cannot be described by Euclidean geometry [10]. After discovering this phenomenon, Mandelbrot proposed fractal geometry to make up the limitation of Euclidean geometry in characterization fractal objects [11,12]. Fractal dimension (FD) is the most important parameter of fractal geometry for it reveals the hierarchical relationship of fractal objects at different scales.

The topography of surface leads to imperfect contact which making the real contact area only a fraction of the nominal area [13,14]. Characterization of rough surface topography is an important step in studying contact problems of two rough surfaces. Traditionally, a series of statistical parameters are used by surface topography characterization methods [15]. For instance, arithmetic mean deviation of the evaluated profile R_a, root mean squared of the evaluated profile R_q, maximum height of the evaluated profile R_z, etc. However, researchers found that with the same parameter R_a value, the topographies of two engineering surfaces may be significantly different [5,16,17]. Therefore, traditional parameters do not have the ability to distinguish the difference between two engineering surfaces properly. Furthermore, the deviation of a surface from its mean plane is assumed to be a stationary random process in traditional approaches. Thus, the rough surfaces are characterized by statistical parameters such as variances of the height, slope and curvature [18]. But it has be observed that the surface topography is a non-stationary random process [19], which means the statistical parameters are relevant to the sampling length and the instrument resolution. In other words, the statistic parameters are not unique for a particular surface [20,21]. Therefore, it is of great importance to characterize engineering surface using intrinsic parameters like FD which does not depend on the sampling length and the instrument resolution. Fractal geometry comes to make up the limitation of traditional geometry in characterizing rough surfaces by using scale-independent parameters [22]. Furthermore, the information of roughness structure that exhibit the fractal be-

* Corresponding author.
E-mail address: liuyongyb@126.com (Y. Liu).

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havior at all length scales can also be provided by fractal geometry [23].

In general, there are two types of fractal curves, the theoretical fractal curves (or regular fractal curves) and natural fractal curves (or irregular fractal curves) [24]. The theoretical fractal curves, such as Koch curve [25] and Peano curve [26], are generated artificially based on self-similarity. The requirement that the entirety and the locality of the curves are completely similar asked by self-similarity is strictly satisfied. Their FD values can be directly calculated by the generated methods. The natural fractal curves, such as the profile of rolling hills, the coastline that changing frequently and the trajectories of particles, are often encountered in nature. Their self-similarity is approximate self-similarity or established in statistical significance. This kind of fractal curves are also termed as statistical fractal curves. However, their FD values are often unknown. It should be noted that the definition of self-similarity is based on the property of equal magnification in all directions. But many objects in nature have unequal rather than equal scaling in different directions (For example, the fractional Brownian motion (fBm)). They are not self-similar but self-affine. Currently, many methods are put forward to extract the FD value. For example, the yard stick method [27], the R/S method [28], the rectangular cell counting method(RCC) [29], the power spectrum method(PS), the root mean square method(RMS) [30], the variogram method(VM) [31], the structure function method(SF) [32], the wavelet transform method [33] and the three-point sinuosity method(TPS) [24]. However, different researchers reach the opposite conclusions when they study the characterization effects of different methods [34]. Although there are already some published articles aiming at evaluating the performance of different fractal dimension calculation methods [2,31], the methods they studied are either infrequently used or have been proved ineffective. Moreover, some new methods proposed in recent years are not included in the evaluation. The calculation accuracy and stability of four commonly used methods—the RCC, the PS, the Roughness-length method and the VM were compared and analyzed by Zhang etc. in article [35]. Their research results are conductive to the development of fractal theory and surface characterization. However, they did not identify the scaling region of the profile curves before calculating the FD value, which would affect the accuracy and objectivity of the calculation results. Therefore, the main purpose of this paper it to evaluate the performance of four commonly used or recently proposed methods, that is the SF, the RMS, the VM and the TPS, from three aspects of fitting accuracy, calculation accuracy and computational stability.

2. Methods

2.1. Weierstrass–Mandelbrot function

Because of its solid theoretical basis, Weierstrass–Mandelbrot (W-M) function is widely used to generate surface profiles [35]. It is given as [1]

$$z(x) = G^{(D-1)} \sum_{n=n_l}^{\infty} \frac{\cos 2\pi\gamma^n x}{\gamma^{(2-D)n}}; \quad 1 < D < 2; \quad \gamma > 1 \tag{1}$$

where $z(x)$ is the surface profile height at the lateral distance x, D is the FD value; G is the characteristic length scale of a surface and is constant, γ^n is the discrete frequency spectrum of the surface roughness and corresponds to the reciprocal of the wavelength of roughness as $\gamma^n = \frac{1}{\lambda^n}$, n_l corresponds to the low cut-off frequency of the profile. It is found that $\gamma = 1.5$ is a suitable value for high spectral density and for phase randomization [36]. Nine W-M function based profiles are created. Their FD values are from 1.1 to 1.9 with an increment of 0.1. The parameters are set as $G = 10^{-8}$, $\gamma = 1.5$, $x = 0.01 \sim 10$. Due to the space limitation, only four profiles, with the FD values of 1.2, 1.4, 1.6, 1.8 respectively, are shown in Fig. 1. It can be seen that the complexity of the W-M profile increases as the FD value increases.

2.2. Methods for calculating FD value

2.2.1. Structure function method

For a fractal curve, the expression of structure function is [37]

$$S(\tau) = \left\langle [z(x+\tau) - z(x)]^2 \right\rangle = \int_{-\infty}^{+\infty} P(\omega)(e^{j\omega\tau} - 1)d\omega \tag{2}$$

where τ is the arbitrary increment of x. $< >$ implies spatial average. $P(\omega)$ is the power spectrum of W-M function, and is given as [1]

$$\hat{P}(\omega) = \frac{G^{2(D-1)}}{2} \sum_{n=n_l}^{\infty} \frac{\delta(\omega - \gamma^n)}{\gamma^{(4-2D)n}} \tag{3}$$

where $\delta(x)$ is the Dirac-delta function. The discrete W-M power spectrum can be approximated by a continuous spectrum [1]

$$P(\omega) = \frac{G^{2(D-1)}}{2\ln\gamma} \frac{1}{\omega^{5-2D}} \tag{4}$$

Substituting Eq. (4) into Eq. (2) and integrating it:

$$S(\tau) = \left\langle [z(x+\tau) - z(x)]^2 \right\rangle = \frac{\Gamma(2D-3)\sin[(2D-3)\pi/2]}{(4-2D)\ln\gamma} G^{2(D-1)}\tau^{4-2D} = C_{SF}\tau^{4-2D} \tag{5}$$

where Γ is the second type Euler Gamma function, $C_{SF} = \frac{\Gamma(2D-3)\sin[(2D-3)\pi/2]}{(4-2D)\ln\gamma} G^{2(D-1)}$.

Using log algorithm to Eq. (5) yields

$$\lg[S(\tau)] = \lg(C_{SF}) + (4-2D)\lg(\tau) \tag{6}$$

From Eq. (6) we can see that there is a linear relationship between $\lg[S(\tau)]$ and $\lg(\tau)$. The slop of the log-log plot can be calculated by the least-squares method. Thus, D can be obtained from the conversion relation between D and the slop k

$$D = (4-k)/2 \tag{7}$$

In the discrete domain, supposing that the digitized data are $z(x_i) = z_i \ (i = 1, 2, \cdots, N)$. Let $\tau = n\Delta t \ (n = 1, 2, 3, \cdots)$ in Eq. (2) and the structure function is yielded as

$$S(\tau) = \left\langle [z(x+\tau) - z(x)]^2 \right\rangle = \frac{1}{N-n} \sum_{i=1}^{N-n} (z_{i+n} - z_i)^2 \tag{8}$$

2.2.2. Root mean square method

Within a certain sampling length $L(\omega_l = 1/L)$, the root mean square of heights for a fractal curve is [1,32]

$$\sigma^2 = \left\langle z(x)^2 \right\rangle = \int_{\omega_l}^{\omega_h} S(\omega)d\omega = \frac{G^{2(D-1)}}{4-2D}\left(\frac{1}{\omega_l^{4-2D}} - \frac{1}{\omega_h^{4-2D}}\right) \tag{9}$$

where ω_l is the low angular frequency and ω_h is the high angular frequency. They are related to the sampling length and sampling interval, respectively. When ω_h is far larger than ω_l, Eq. (9) can be expressed as

$$\sigma^2 = \frac{G^{2(D-1)}}{4-2D} \frac{1}{\omega_l^{4-2D}} = \frac{G^{2(D-1)}}{4-2D} L^{4-2D} = C_{RMS}L^{4-2D} \tag{10}$$

where $C_{RMS} = \frac{G^{2(D-1)}}{4-2D}$.

Using log algorithm to Eq. (10) yields

$$\lg(\sigma) = \lg(C_{RMS})/2 + (2-D)\lg(L) \tag{11}$$

Therefore, D can also be obtain from the relationship between D and the slop k

$$D = 2 - k \tag{12}$$