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Hybrid consensus for averager–copier–voter networks with non-rational agents

Yilun Shang

School of Mathematical Sciences, Tongji University, Shanghai 200092, China

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ABSTRACT

For many social dynamical systems with heterogenous communicating components there exist non-rational agents, whose full profile (such as location and number) is not accessible to the normal agents a priori, posing threats to the group goal of the community. Here we demonstrate how to provide resilience against such non-cooperative behaviors in opinion dynamics. We focus in particular on the consensus of a hybrid network consisting of continuous-valued averager, copier agents and discrete-valued voter agents, where the averagers average the opinions of their neighbors and their own deterministically, while copiers and voters update their opinions following some stochastic strategies. Based upon a filtering strategy which removes some fixed number of opinion values, we establish varied necessary and sufficient conditions for the hybrid opinion network to reach consensus in mean in the presence of globally and locally bounded non-rational agents. The communication topologies are modeled as directed fixed as well as time-dependent robust networks. Although our results are shown to be irrespective of the proportion of the averager, copier, and voters, we find that the existence of voters has distinct influence on the evolution and consensus value of the negotiation process.

1. Introduction

Decision-making in today's networked complex systems requires coordination of a group of heterogenous agents, which may comprise cyber, physical, and human elements. These agents naturally engage in consensus-building on certain quantities of interest, such as heading of autonomous vehicles, temperature of the environment, and opinion of the cohort, through local interaction with their neighbors. For example, in the setting of social networks, interacting agents can influence each other and gradually form common opinions by ignoring minority opinions and allowing opinion differences [1].

A number of physical models have been developed to explore human opinion propagation despite the challenge of describing and evaluating the macroscopic collective behaviors involving physiological and psychological factors. In opinion dynamics, each agent maintains an opinion, i.e., state, which can be a continuous or a discrete quantity. In the discrete case, binary opinion models have dominated research, mostly in physics literature, due to their marked analogy with spin systems. The voter model [2], the Sznajd model [3] and the Galam majority-rule model [4] are well-known examples of discrete opinion dynamics models. There are also situations where the opinion of an agent should be expressable in real and may vary smoothly between the extremes. Examples include attitudes, prices, or predictions about macroeconomic variables. Among continuous opinion dynamics models, the Deffuant–Weisbuch model [5] and the Hegselmann–Krause model [6] have attracted significant attention in sociophysics. The dependence of the final opinion distribution on the confidence bound in Deffuant–Weisbuch model, for example, has been studied in directed small-world networks [7]. The effect of self-affirmation on the opinion formation relating to the phase transition of the opinion configuration has also been examined for directed small-world networks [8]. It is noteworthy that most current models of opinion dynamics and more generally, distributed multi-agent coordination [9], are concerned with either continuous or discrete opinion (state) spaces, but not a mix of them.

In practice, discrete-valued and continuous-valued players or opinions often exist concurrently: examples include discrete action selection based on fusion of sensed quantities [10], heterogenous cyber-physical systems [11], and political elections, where some participants have binary opinions while others are able to hold graded ones [12]. In the recent work [13], the author interprets a class of network dynamics with hybrid discrete-valued and continuous-valued opinions as averager–copier–voter models [14]. In this endeavor, the network is composed of three types of normal agents, which update their states in different fashions, namely,

E-mail address: shylmath@hotmail.com

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averaging, stochastic copying, and stochastic binary voting. Based upon moment methods and stochastic stability theory, it is proved that consensus can be achieved in mean-square sense if the agents perform independent random walks over a spatial network that is connected and non-bipartite [13]. This result is remarkable in that rigorous mathematical analyses for opinion dynamics have been so far essentially overlooked, with the notable exceptions of discrete-valued voter model (e.g. [15]) and continuous-valued Deffuant–Weisbuch model (e.g. [16–18]) due to the complexity of the involved dynamical processes.

An important challenge of decision-making in social networks is that the community contains non-rational behaviors that prevent the group of normal agents from achieving their goal [19]. For example, there can be stubborn agents, who influence others but do not adjust their opinions, or malicious agents, who deliberately change their opinions with the goal of manipulating the performance of entire network. In this paper, we aim to continue the line of research [13] by considering the averager–copier–voter networks with non-rational agents over fixed and time-varying networks. We incorporate a filtering strategy into the rules of averager, copier, and voter agents such that consensus can be achieved in the sense of convergence in mean. Previous works regarding opinion dynamics with non-rational agents (mostly stubborn ones; see e.g. [20–23]) generally characterize how equilibrium depends on the location and number of stubborn agents and show that the presence of stubborn agents precludes convergence to consensus. On the other hand, opinion consensus algorithms based on leadership concept have been studied in [24], where adding minimum edges may lead the network to some established target, which is desirable in networks of firms and administrations. Here, we deal with essentially an inverse problem and present necessary and sufficient conditions for consensus when the identities and actual number of non-rational agents remain unknown to the normal agents. Our method is of practical significance in that it not only contributes to understanding the influence of general adversarial behaviors (from stubborn to highly malicious ones) in opinion formation within a community but offers effective ways to cope with them when each normal agent in the network only communicates with its neighbors, i.e., in a purely distributed manner.

It is worth mentioning that there are some relevant works similar in concept to our current work in robotics and control communities. The work [25,26] introduced a linear consensus protocol, termed Weighted-Mean Subsequence Reduced algorithm, which leads the states of cooperative nodes to an agreement in a multi-agent system asymptotically in the presence of attackers. However, these studies do not capture hybrid dynamics and the models considered therein are deterministic. Also in analogy with the protocols considered here, hybrid consensus formation has been studied in [27,28] focusing on a hybrid of discrete-time and continuous-time agent dynamics, where only continuous-valued states are considered.

2. System description for the averager–copier–voter model

We consider a time-dependent directed graph of order n, denoted by $\mathcal{G}(t)=(\mathcal{V},\mathcal{E}(t))$, where $\mathcal{V}=\{v_1,\cdots,v_n\}$ is the node set representing the agents in the network, and $\mathcal{E}(t)\subseteq\mathcal{V}\times\mathcal{V}$ is the directed edge set at time $t\in\mathbb{N}$. Here, $\mathbb{N}$ means the set of non-negative integers. Assume a partition of the node set, $\mathcal{V}=\mathcal{N}\cup\mathcal{M}$, where $\mathcal{N}$ is the set of normal agents and $\mathcal{M}$ is the set of non-rational agents which is unknown a priori to the normal ones. An edge (v_i, v_j) captures the information flow from agent v_i to agent v_j. The neighborhood of agent v_i at time t is defined by $\mathcal{J}_i(t)=\{v_j:(v_j,v_i)\in\mathcal{E}(t)\}$. When considering fixed networks, we will omit the dependence on t accordingly in the above notations. When the meaning is clear from the context, we also suppress t in time-dependent networks for simplicity.

Let $x_i(t)$ represent the value, i.e., opinion, of agent v_i at time step $t\in\mathbb{N}$. Denote by $x_j^i(t)$ the value sent from agent v_j to agent v_i at time t, and assume $x_j^i(t)=x_j(t)$ for $v_j\in\mathcal{N}$. We further assume that $\mathcal{N}=\mathcal{N}_a\cup\mathcal{N}_c\cup\mathcal{N}_v$, where $\mathcal{N}_a,\mathcal{N}_c$, and $\mathcal{N}_v$ represents the averager, copier, and voter agents, respectively. Fix $R\in\mathbb{N}$ (which is related to the number of opinion values to be removed or ignored when performing our strategy; see below) and assume $x_i(0)\in[0,1]$ for all $v_i\in\mathcal{N}$. We do not impose any assumption on the initial values of non-rational agents. The normal agents are proposed to update their opinions according to the following 3-step strategy, executed at each time step $t\in\mathbb{N}$:

(1) Each normal agent $v_i\in\mathcal{N}$ obtains the values $x_j^i(t)$ of its neighbors, and creates a sorted list for $\{x_j^i(t)\}_{v_j\in\mathcal{J}_i(t)}$ from largest to smallest.

(2) The largest R values that are strictly larger than $x_i(t)$ in this list are removed (if there are fewer than R larger values than $x_i(t)$, all of those values are removed). The similar removal process is applied to the smaller values. The set of nodes that are removed by agent v_i at time t is denoted by $\mathcal{R}_i(t)$.

(3) For averager $v_i\in\mathcal{N}_a$:

$$x_i(t+1)=\sum_{v_j\in(\mathcal{J}_i(t)\cup\{v_i\})\backslash\mathcal{R}_i(t)} w_{ij}(t)x_j^i(t), \tag{1}$$

where $\{w_{ij}(t)\}$ are weights satisfying (i) $w_{ij}(t)=0$ if $v_j\notin\mathcal{J}_i(t)\cup\{v_i\}$, (ii) there exists a constant $\alpha\in(0,1)$ independent of t, such that $w_{ij}(t)\ge\alpha>0$ for any $v_j\in(\mathcal{J}_i(t)\cup\{v_i\})\backslash\mathcal{R}_i(t)$, and (iii) $\sum_{v_j\in(\mathcal{J}_i(t)\cup\{v_i\})\backslash\mathcal{R}_i(t)} w_{ij}(t)=1$. Note that the weights $\{w_{ij}(t)\}$ can be arbitrarily chosen as long as the conditions (i)–(iii) hold. This distributed averaging protocol maintains continuous-valued opinions and calculates a continuous-valued function to determined the next opinions. It has been widely adopted in cooperative coordinations [13,20,27], which models the agents desiring to match their opinions with neighbors' and having good computation capability.

For copier $v_i\in\mathcal{N}_c$: First, choose an agent v_j with probability $w_{ij}(t)$, and then, set $x_i(t+1)=x_j^i(t)$, where $\{w_{ij}(t)\}$ satisfy the above three conditions (i)–(iii). Here, each copier agent randomly selects a determining agent (namely, a neighbor or itself), and copies its opinion at the next time step. By doing so, the copier agents maintain a continuous-valued opinion but adjust this opinion via a discrete selection process. A copier can be thought of as one who does not have its own idea and is randomly influenced by others.

For voter $v_i\in\mathcal{N}_v$: First, similarly as for a copier, choose an agent v_j with probability $w_{ij}(t)$, and then take $x_i(t+1)=1$ with probability $x_j^i(t)$ and 0 with probability $1-x_j^i(t)$, where $\{w_{ij}(t)\}$ satisfy the above three conditions (i)–(iii). We remark that $x_j^i(t)\in[0,1]$ for the models we are interested here (see below). Clearly, each voter is stochastically influenced by its neighbors' opinions but maintains a binary opinion (0 or 1) itself via a discrete update procedure. The voters have some pre-determined ideas and use their neighbors' opinions to make their own choice.

Recent empirical studies in social learning have also revealed that the transmission of behaviors can happen in different ways of copying, and that the specific copying mechanisms involved in decision making can impact the resulting diffusion dynamics at population level [29]. The three individual behaviors introduced above, namely, averager, copier, and voter, can be roughly viewed as using "copying from the majority", "copying from the random one", and "copying from the best one", respectively. On an intuitive level, these actions tend to produce consensus behaviors.