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Tolerance-based punishment and cooperation in spatial public goods game

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ABSTRACT

Punishment, as a remarkable way, has been proposed to explain the emergence and persistence of cooperation in the human species. Inspired by the fact that people have a certain tolerance for free-riders before punishment, therefore, we study the effect of tolerance-based punishment on the evolution of cooperation in spatial public goods game. Cooperators punish defectors on the basis of the tolerance threshold during the evolutionary process and have to bear the relevant costs of sanction subsequently. Different from previous works, the new mechanism can reduce the frequency of punishing by controlling the tolerance for punishment. We find that this mechanism can lead to synergistic effects, and it can stabilize the circumstance of full cooperation under adverse conditions. By means of analysis of the emergence of cooperative clusters, we demonstrate that the tolerance-based punishment can promote cooperation through enhancing spatial reciprocity. In addition, the readiness of cooperation increases obviously by adjusting this kind of punishment. Our work extends the form of punishment in the evolution of spatial public goods game and the results are conducive to a better understanding of punishment.

1. Introduction

Although competition and natural selection among species drive their evolution and theoretically bring more benefits to defectors, the emergence and maintenance of cooperation among unrelated individuals are still abundant, ranging from biological spheres to microorganism groups and complex social systems in the real world [1–4]. Consequently, many scientific researchers from myriad fields pay attention to these cooperative phenomena and try to understand them. They usually resort to the most prevailing framework of evolutionary game theory within population dynamics to uncover the intrinsic mystery of cooperation [5,6]. Up to now, there is no doubt that the prisoner's dilemma game [7–9] is the most popular model to explore the potential supporting mechanism for cooperation of pairwise interactions under this framework. However, the N-person prisoner's dilemma game, namely, the so called public goods game (PGG) [10,11], is the metaphor to study the evolution of cooperative behavior for multi-individual interactions. Clearly, dealing with the environmental issues (overgrazing and overfishing) or politics and so forth, the PGG is more reasonable to observe and explain the sustainability of cooperation [12].

In a traditional PGG, all agents decide whether to cooperate or not simultaneously. It is used to assume that a cooperator ($s_i = C$) contributes a fixed cost ($c = 1$) to the public pool and a defector ($s_i = D$) pays nothing. Afterwards, the sum of contributions in the public pool multiplies the synergy factor r ($1 < r < N$, N is the size of the group). The resulting product is then allocated among whole participators within the system equally, irrespective of what strategy they have selected. Defectors are able to obtain the same payoff with cooperators without efforts and dominate the whole group ultimately. Apparently, defection is the rational Nash equilibrium for this game. No one would prefer to invest in common pool if the above description happens. That is to say, although all individuals' cooperation can maximize whole payoff, defection is always the optimal choice no matter what the others' strategies are in this model, which is the well known social dilemma [13].

Over the past decades, it has been proved that defection can be effectively suppressed in this social dilemma with the help of a variety of factors and mechanisms, i.e. reward [14,15], reputation [16], heterogeneous activity [17,18], social diversity [19–21], structured populations [22], punishment [23,24], win-stay lose-shift [25], and so on [26–30]. Among them, spatial structure as a paradigm is one of the most conspicuous works and has been intensively studied [7]. In spatial game, individuals are fixed on the nodes of the lattice. Player i could interact with his neighbors only. In addition, inspired by this prominent work, various topologies has been studied to illustrate how cooperation evolves [31–36].

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What triggers our interest is the punishment mechanism in PGG, which has been proved to be an effective method to facilitate cooperation theoretically and experimentally [37–43]. Moreover, punishment is frequently observed in animal society [44]. Usually, the punishers' sanctions are considered to be costly. Therefore, punishers must bear the cost of punishment and those who are punished suffer a fine. Obviously, this approach will diminish the overall payoff of both sides. In addition, the implemented patterns of punishment rely on the evolutionary process, environment and individual strategies. As a consequence, the influence of punishment could be revealed in completely different ways in structured populations. Although punishment is costly and punishers do not get any benefit in PGG experiment, it has been confirmed that cooperation will flourish if altruistic punishment is possible [45]. Recently, pool punishment is also emphasized as an effective method for promoting the evolution of cooperation [46–48]. It is shown that punishment is dependent on emotion and tolerance [49]. This illustrates that punishment may have a quantitative change for a period of time. In reality, it is impossible that cooperators prefer to punish selfish players permanently even if punishment is widespread. Most humans are inclined to cooperate visibly [50]. This is because the execution of punishment is pretty costly and abstinence is unavoidable [51]. Moreover, a tolerant strategy is introduced and the diversity of tolerance can enhance cooperation effectively [52]. Tolerance-based punishment combining with investment is also studied [53]. Motivated by these observations, here we drop the assumption that cooperators may punish defectors immediately. In contrast, we introduce a threshold for punishment, which means that punishment will be implemented after a tolerance stage. We wonder how cooperation fares when tolerance-based punishment is considered in the spatial public goods game. In other words, once defectors break the tolerance of cooperators, cooperators will punish the corresponding players. And cooperators have to bear the cost.

The rest of this paper is organized as follows. Section 2 shows the basic model. And then, numerous simulation results are presented in Section 3 to discuss the effect of tolerance-based punishment in promoting cooperation. Section 4 concludes this paper.

2. Methods

In present work, the PGG is considered on a $L \times L$ regular lattice with periodic boundary conditions. The agents occupy the vertices of the lattice. Individual i takes part in $G = 5$ overlapping groups centered at i and its $G - 1$ neighbors, respectively. As a consequence, each player pertains to $g = 1, \ldots, G$ different small systems. Originally, every individual is arranged as a cooperator ($s_i = C = 1$) or a defector ($s_i = D = 0$) with the same probability. As mentioned earlier, cooperators must pay a same cost $c = 1$ to the public pool, but defectors pay nothing. Thereafter, the sum of contributions in the group multiplies the synergy factor r ($1 < r < G$) and the amount will be distributed equally in the group, irrespective of whether they have invested or not. Therefore, the player i obtains payoff P_i^g from a group g expressed by the following mathematical formula

$$P_i^g = \begin{cases} \frac{r \cdot \sum_{j \in g} s_j \cdot c}{k_i + 1} - s_i \cdot c, & \text{if} \quad s_i = C, \\ \frac{r \cdot \sum_{j \in g} s_j \cdot c}{k_i + 1}, & \text{if} \quad s_i = D, \end{cases} \tag{1}$$

where j is one of the members in group g and k_i is the degree of player i. Since each player participates in $G = 5$ PGGs, accordingly, the total payoff P_i of player i is

$$P_i = \sum_{g=1}^{G} P_i^g. \tag{2}$$

In addition, parameter t_i is arranged to record the successive times of defection for player i and M_i represents the tolerance threshold. Here we assume that M_i is same for all agents. Hereafter it is represented only by M. Of particular reputation, t_i is also permitted to change according to the following agreement. All players possess the same parameter $t_i = 0$ to avoid preferential influence before the game. However, t_i will plus 1 if player i chooses defection. Otherwise, it is always zero. That is to say, t_i will jump to zero once player i changes from defection to cooperation. Defector i will be punished with a fine α in the group if $t_i > M$, and all cooperators who participate in the punishing share the corresponding costs equally at the same time. Once player i satisfies the above situation, he would calculate his payoff according to the following equation:

$$P_i^g = \begin{cases} \frac{r \cdot \sum_{j \in g} s_j \cdot c}{k_i + 1} - s_i \cdot c - \frac{n_d \cdot \alpha}{n_c}, & \text{if} \quad s_i = C \quad \text{and} \quad t_i > M, \\ \frac{r \cdot \sum_{i \in g} s_j \cdot c}{k_i + 1} - \alpha, & \text{if} \quad s_i = D \quad \text{and} \quad t_i > M. \end{cases} \tag{3}$$

Here n_c is the number of cooperators who carry out the punishment simultaneously in the group centered at player i, and n_d is the number of defectors who are beyond the limit of the partners' tolerance M. What needs to be emphasized is that cooperators who decide to punish bear the same cost. In two extreme cases, it is worth pointing out that Eq. (3) tends to Eq. (1) when M tends to infinity. On the contrary, $M \to 0$ indicates that the punishment will work immediately like previous works. Undoubtedly, such a punishment will certainly lead to a heterogeneous state of players. Besides, Anna et al.find that those people who get the highest payoff tend not to adopt costly punishment [54]. This shows that this mechanism has realistic bases.

Obviously, it can distinguish free-riders from the whole system by means of this simple mechanism. Compared with the traditional case, individuals will punish free-riders according to the tolerance boundary in the new model. This would lead to heterogeneity of payoff, which is the main characteristic of current models. For example, defectors have to suffer a lose in either reputation or property if they are always exploiting others in reality. The random initial strategy distribution is the start of Monte Carlo (MC) simulations. Subsequently, it transfers to the strategy updating stage. Player i will adopt the strategy of player j who was chosen randomly according to the probability H:

$$H = \frac{1}{1 + \exp[(P_i - P_j)/K]}, \tag{4}$$

where K represents the selection intensity [55–58]. It is noted that each player has a chance to adopt one of his neighbors' strategies once on average during one full MC simulation.

Results of MC simulations in the *RESULTS* section are carried out on population comprising 200×200 to 400×400 individuals. We study the key quantity of cooperator density ρ_c in the equilibrium state. The important quantity fraction of cooperators of each data point is computed by averaging the last 10,000 steps of the total 1×10^5 MC steps. In addition, the final results are averaged over 10 independent runs to guarantee the accuracy and overcome the initial distribution.

3. Results

First of all, to provide a comprehensive view, we show the cooperators' evolutionary fate with respect to the combined points of parameter α and M in Fig. 1. It is remarkable that $r = 3.7$ is a relatively low value of the enhancement factor in the spatial PGG. However, the monotonic dependence can still be seen in such a situation. Obviously, the lower right corner ($\rho_c = 1$) and the upper left corner ($\rho_c = 0$) are the pure strategy states, and the central