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Perturbed soliton and director deformation in a ferronematic liquid crystal

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ABSTRACT

The nonlinear molecular deformation of the ferronematic liquid crystal in the presence of external applied magnetic field intensity is investigated in view of solitons for the director axis. The Frank's free energy density of the nematic liquid crystal comprising the basic elastic deformations, molecular deformation associated with the nematic molecules and the suspended ferromagnetic particles and their interactions with magnetic field intensity is deduced to a sine-Gordon like equation using the classical Euler–Lagrange's equation. Using the small angle approximation we establish the Ginzburg–Landau (GL) equation and a class of solutions are obtained. In the normal condition of large angle oscillation of the director axis, we constructed a damped sine-Gordon (sG) equation with the additional perturbation appears in the form of cosine function. The sG equation is solved using numerical simulation and kink excitations were obtained as the molecular deformation for the case of constant damping and distorted kink to a planar configuration transition as we increase the damping.

1. Introduction

Molecular deformation in nematic liquid crystal (NLC) is an interesting object of study in the community of nonlinear science [1–3]. Due to its preferred geometrical structures owing to the elastic constants leading to the splay, bend and twist type deformations the average molecular orientations of the NLC is represented in the form of the director **n**. The inclusion of the external fields and the forces exerted by the boundaries of the nematic container leads to the imbalance of the NLC molecules thereby creating an effective torque. The governing dynamical equation can be framed by balancing the effective torque induced by the viscous field with that of the elastic and external fields. In this context recently the impact of the magnetic field associated with the NLC is widely studied because doping the ferromagnetic nanoparticles in the nematic system can enrich molecular excitations because all liquid crystal materials are completely diamagnetic. The idea of dispersing the ferroparticles in nematic was first proposed by Brochard and de Gennes in 1970 [4]. In the absence of ferroparticles the nematic liquid crystal material requires magnetic field as high as 1 kOe in order to achieve appreciable molecular deformations. However, for ferronematic all it requires is just a minimum field strength of 10 Oe to control their orientations. In the ferronematic liquid crystal the magneto-optical and orientational effect is highly influenced by the applied magnetic field [5]. The magneto-optical response is affected by the coupling energy between nematic molecules and suspended ferromagnetic particles. There exist a threshold value of the coupling energy below which no significant effect in the dynamics was observed. For a strong coupling between the molecules the system undergoes the Freedericksz transition i.e. the initial uniform state is changed to a non-uniform state and beyond this transition the director and the magnetization rotates uniformly in the field direction. There exist another interesting transition for the weak coupling energy where uniform compensated phase corresponding to the director-magnetization parallel to the orientation axis at the boundaries change to non-uniform phase and uniform saturation phase corresponding the director parallel to the orientation axis and magnetization align parallel to the applied field to a non-uniform saturated phase. In view of the electromagnetic field effects ferronematic liquid crystal act as a perfect polarizer for the polarization grating device [6]. It is found that the critical thickness of the polarizing device is inversely proportional to the dimensions of the suspended ferromagnetic particles and using the thicker cells we can manipulate the device performance with smaller applied magnetic fields. The critical thickness and the threshold field for the Freedericksz transition can be reduced by proper construction of the boundary walls for the weak anchoring. The results reported

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so far in the context of ferronematic materials is primarily focused on phase transition and its implication in constructing magneto-optical devices but however the soliton excitations in ferronematic liquid crystal is another interesting study which is not reported earlier. In this paper we present a systematic formulation of the nonlinear dynamics of the ferronematic liquid crystal in the frame of solitons. The paper is organized as follows. In Section 2, we formulate the relevant dynamical equation by considering the free energy density along with the Euler-Lagrange equations. In Section 3, class of solution were reported for the Ginzburg–Landau equation as a deduction of the sine-Gordon like equation. The perturbed sine-Gordon equation is derived in Section 4 and the numerical simulation performed on the perturbed sine-Gordon equation and the effect of full nonlinearity is discussed in Section 5. Finally, the conclusions are presented in Section 6.

2. Molecular fields and dynamics

We consider ferromagnetic particles that are dispersed into the nematic liquid crystal. The dispersed ferroparticles generates a strong agglomeration which leads to a magnetic particle string formation (Fig. 1). We focus on one such magnetic string and frame the dynamics. The free energy density of the ferronematic liquid crystal for the director orientation $\theta(z, t)$ about the z direction reads

$$F = \frac{1}{2}K_1\cos^2\theta\left(\frac{\partial\theta}{\partial z}\right)^2 + \frac{1}{2}K_3\sin^2\theta\left(\frac{\partial\theta}{\partial z}\right)^2 - \frac{1}{2}p\mu_0^{-1}\chi_a B_a^2\sin^2\theta - \frac{1}{2}p\mu_0^{-1}\chi_F B_a^2\sin^2\beta + w\cos^2(\theta-\beta). \quad (1)$$

In Eq. (1), the first two terms on the left side corresponds to the splay and bend type deformation with elastic constants K_1, K_3, χ_a, χ_F are the magnetic anisotropies of the nematic and the ferromagnetic nanoparticles, p is the ferroparticles concentration in the mixture (volumetric fraction), B_a is the magnetic field intensity directed along the z direction, μ_0 is the magnetic permeability of vacuum, the factor w can be written using the anchoring energy w_0, the anchoring angle α and the radius of the magnetic particle string R [7]

$$w = \frac{w_0 p}{R}(1 - 3\cos^2\alpha). \quad (2)$$

The nonlinear molecular deformations in the ferronematic matrix can be investigated by minimizing the free energy through the Euler–Lagrange equations for the dynamical variables θ and β, and it is given by

$$h_\theta = -\frac{\partial F}{\partial\theta} + \frac{\partial}{\partial z}\left(\frac{\partial F}{\partial\theta_z}\right) = 0, \quad (3)$$

$$h_\beta = -\frac{\partial F}{\partial\beta} + \frac{\partial}{\partial z}\left(\frac{\partial F}{\partial\beta_z}\right) = 0. \quad (4)$$

We substitute the free energy density (1) in Euler–Lagrange Eq. (4), yields

$$-B_a^2\chi_F\mu_0^{-1}p\sin\beta\cos\beta + 2w\cos(\theta-\beta)\sin(\theta-\beta) = 0, \quad (5)$$

which in small angle approximation for β, leads to [8]

$$\beta = \left(\frac{2w}{2w + B_a^2\chi_F\mu_0^{-1}p}\right)\theta \quad (6)$$

Using the Eq. (3) and reporting all the free energies into the same, the effective molecular field can be written as

$$h_{eff} = \left[K_1\cos^2\theta + K_3\sin^2\theta\right]\left(\frac{\partial^2\theta}{\partial z^2}\right) + \left[\frac{1}{2}p\mu_0^{-1}\chi_a B_a^2 + \frac{1}{2}(K_3-K_1)\left(\frac{\partial\theta}{\partial z}\right)^2\right]\sin 2\theta - A_0\theta\cos 2\theta. \quad (7)$$

In addition to the effective molecular field the nematic molecule is always driven by the viscous field due to the rotational effect along with the coupling of director axis with that of the fluid motion. In many cases the effect of director coupling with the fluid motion can be neglected in the limit of low fluid velocity. The viscous field is given by $\gamma_1\frac{\partial\theta}{\partial t}$. The dynamical equation can be written by balancing the molecular field with that of the viscous field one can write the equation as

$$\gamma_1\frac{\partial\theta}{\partial t} = \left[K_1\cos^2\theta + K_3\sin^2\theta\right]\left(\frac{\partial^2\theta}{\partial z^2}\right) - A_0\theta\cos 2\theta + \left[\frac{1}{2}p\mu_0^{-1}\chi_a B_a^2 + \frac{1}{2}(K_3-K_1)\left(\frac{\partial\theta}{\partial z}\right)^2\right]\sin 2\theta. \quad (8)$$

The coefficient γ_1 is the viscosity coefficient associated with the pure rotational motion of the director axis. The coefficient $A_0 = \left(\frac{4w^2}{2w+B_a^2\chi_F\mu_0^{-1}p}\right)$. Thus, the molecular excitation in the ferronematics liquid crystal is governed by a sine-Gordon-like equation.

3. Solution through Ginzburg–Landau equation

The sine-Gordon like equation obtained in the previous section can be reduced to a more standard nonlinear equation. We apply the transformation $\theta = \frac{3}{2}\theta'$ and drop the prime, we have

$$\gamma_1\frac{\partial\theta}{\partial t} = \frac{1}{2}\left[(K_1+K_3) - (K_1-K_3)\cos 3\theta\right]\left(\frac{\partial^2\theta}{\partial z^2}\right) + \left[\frac{3}{4}(K_1-K_2)\left(\frac{\partial\theta}{\partial z}\right)^2 + \frac{1}{3}p\mu_0^{-1}\chi_a B_a^2\right](3\sin\theta - 4\sin^3\theta) - A_0\theta\cos 3\theta. \quad (9)$$

In the limit of small angle oscillation, the deviation of director from the equilibrium state is assumed to be very small, i.e. $\theta(z) << 1$ and under this small angle and one elastic constant approximation $(K_1 = K_3 = K)$ the dynamical Eq. (9) leads to

$$\gamma_1\frac{\partial\theta}{\partial t} - K\frac{\partial^2\theta}{\partial z^2} - \frac{1}{2}B_a^2\mu_0^{-1}p\chi_a\left(1 - \frac{4}{3}\theta^2\right)\theta - A_0\theta = 0. \quad (10)$$

The reorientation dynamics of the director axis can be understood by solving Eq. (10) using a multiscale expansion procedure [9] for the solution in the form

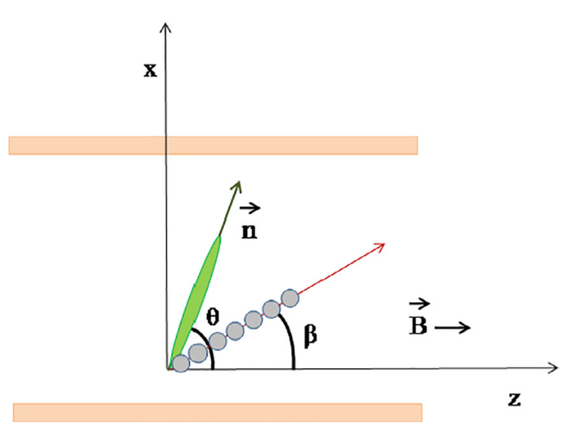

Fig. 1. The ferronematic liquid crystal with director and magnetic particle string orientation.