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Fractal and multifractal analysis of the rise of oxygen in Earth's early atmosphere

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ABSTRACT

The rise of oxygen in Earth's atmosphere that occurred 2.4–2.2 billion years ago is known as the Earth's Great Oxidation, and its impact on the development of life on Earth has been profound. Thereafter, the increase in Earth's oxygen level persisted, though at a more gradual pace. The proposed underlying mathematical models for these processes are based on physical parameters whose values are currently not well-established owing to uncertainties in geological and biological data. In this paper, a previously developed model of Earth's atmosphere is modified by adding different strengths of noise to account for the parameters' uncertainties. The effects of the noise on the time variations of oxygen, carbon and methane for the early Earth are investigated by using fractal and multifractal analysis. We show that the time variations following the Great Oxidation cannot properly be described by a single fractal dimension because they exhibit multifractal characteristics. The obtained results demonstrate that the time series as obtained exhibit multifractality caused by long-range time correlations.

1. Introduction

Fractal methods are well-suited for describing self-similarity, jaggedness and correlation of data sets. Typically, the data is characterized by the so-called fractal dimension, which can be evaluated by finding how the data fills its embedding space [1]. The data can be plotted and then the jaggedness of such graph can be compared to a straight line, leading to the box counting method or correlation integral methods. Another way is to treat the data as discrete and to compare it to uncorrelated noise, like in the well-known Hurst exponent method [2]. The fractal methods are generalized by multifractal methods that have different scaling moments for different moments or for different magnitudes of fluctuations. However, if a scaling component depends on the scale, there is a crossover, and the multifractal analysis must be separately applied to ranges of small and large scales. Typically, a singularity spectrum [3,4] is computed to determine multifractal characteristics of time series data.

The fractal and multifractal methods have been extensively applied to different problems in natural sciences, engineering, medicine, and the social sciences [3–10]. However, these methods have only been sparsely applied within the recently established field of astrobiology or its terrestrial counterpart, i.e., biogeology/geobiology, though the study by [11] on exoplanetary life detectability represents a notable exception. In this paper, we pursue another applications, a topic of great importance, namely, the rise of oxygen in Earth's early atmosphere. It consists of the Earth's Great Oxidation, followed by a more gradual increase of Earth's atmospheric oxygen level.

In the following, we describe the problem in detail, and also explain why we chose the fractal and multifractal methods for this study. The Earth's Great Oxidation occurred 2.4–2.2 billion years ago and it had a significant impact on Earth's atmospheric physics and chemistry and, furthermore, entailed profound implications for the evolution of

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life. During this time span, the atmospheric oxygen concentration rose from less than 10^{-5} of the present atmospheric level (PAL) to more than 0.01 PAL and possibly above 0.1 PAL [12–15]. Thereafter, a more gradual increase of the oxygen level occurred, eventually leading to today's level. The rise of oxygen is still a topic of lively discussion owing to its wide ramifications regarding terrestrial and planetary astrobiology; specifically, it is still unclear whether Earth's Great Oxidation occurred relatively smoothly or exhibited big jumps, i.e., akin to a yoyo model[1] (see [18]). Moreover, the entire sequence of oxygen increases, including the increase during the Proterozoic eon after the Great Oxidation, could have occurred relatively smoothly or by exhibiting intricate mathematical structures, which if present can only be uncovered through multifractal analysis.

Information on the rise of oxygen, especially the Great Oxidation, is in part based on studies of atmospheric sulphur chemistry, including analyses of multiple sulphur isotopes, as, e.g., the isotopic ratios between $\Delta^{32}S$, $\Delta^{34}S$, and $\Delta^{36}S$; see work by Farquhar et al. (2000) [19], Ohmoto et al. (2006) [16], Farquhar et al. (2007) [20], and Johnston (2011) [21] (in analogy using isotopic ratios to estimate the age of the Earth [22]). These studies indicate the patterns of increase of early Earth's oxygen levels during different time intervals, including possible fluctuations. Studies providing additional insight into the long-standing debate regarding O_2 in Earth's early atmosphere include the work by Claire et al. (2006) [23], Lyons (2007) [24], Kaufman et al. (2007) [25], Kump (2008) [26], Balk et al. (2009) [27], Frei et al. (2009) [28], Freund et al. (2010) [29], Freund (2011) [30], and Flannery & Walter (2012) [31].

A set of nonlinear equations describing the time evolution of oxygen, methane and carbon in the early Earth was originally proposed and solved by Goldblatt et al. (2006) [32], thereafter GLW06, who encountered bistability in the system equations, which in their model represent the ancient Earth's low-oxygen state ($\lesssim 10^{-5}$ PAL) and the high-oxygen state (5×10^{-3} PAL). Subsequent work by Cuntz et al. (2009) [17], thereafter CRM09, on the nonlinear set of equations further explored this system by replacing the original step function (GLW09) representing the reductant input rate by more realistic functions (i.e., exponential decay function, logistic decay function) with and without Gaussian white noise. Based on the transition stability analysis for the system equations, CRM09 considered a set of non-autonomous, nonlinear differential equations and furthermore inspected the Lyapunov exponents [33–35]. CRM09 found that the equations do not show chaotic behavior and that the rise of oxygen during the Great Oxidation occurred relatively smoothly.

Previous models of Earth's Great Oxidation and the subsequent oxygen increase during the early Proterozoic eon have been based on a set of physical parameters that were determined using the available geological data as summarized by GLW06 and CRM09. However, it is well-known that this set of parameters is not unique because of potentially large uncertainties in the data. Therefore, it is the aim of the present work to expand those previous studies by adopting other sets of parameters and also analyzing the obtained results by using fractal and multifractal techniques. Specifically, we will use the standard Hurst exponent [2] and the fractal dimension related to it [1], as well as the Multifractal Detrended Fluctuation Analysis (MFDFA) [34], which was originally developed for non-stationary time series and required the so-called generalized Hurst exponents for computing a width of singularity spectrum (WSS). Our choice of using the MFDFA is motivated by the fact that our numerical solutions constitute non-stationary time series.

Our studies are performed for different strengths of white Gaussian noise, which is added to the system to account for possible variations in the physical parameters as well as for uncertainties in the adopted values of the parameters. We consider different levels of noise in the system and investigate their effects on the rise of oxygen as well as on the associated time variations of atmospheric methane and carbon in the Earth's surficial environment. The effects of the noise are studied by performing fractal and multifractal analysis of our numerical results. We will be able to demonstrate that no single fractal dimension can be used to describe time variations of oxygen, carbon and methane because they exhibit multifractal characteristics due to long-range correlations of the small and large fluctuations in the time series.

Our paper is structured as follows: in Section 2, we describe the system equations considered in our study, the numerical method of solution, and the methods for performing the fractal and multifractal analysis. In Section 3, we present our results and discussion. Our conclusions are given in Section 4.

2. Formulation and analysis techniques

2.1. Original governing equations

The set of equations was originally given by GLW06 and was subsequently revisited by CRM09. It encompasses a simplified model of Earth's global redox system, representing the atmosphere, ocean, and crust. Concerning the atmosphere and ocean, the number of moles of methane $\mathcal{M}$ (CH_4) and oxygen $\mathcal{O}$ (O_2) are calculated. Furthermore, with respect to the ocean floor, the amount of buried organic carbon $\mathcal{C}$ in the crust is also computed. This leads to the following system of equations:

$$\frac{d\mathcal{M}}{dt} = \frac{1}{2}\Omega_{(O_2)}(N+r) - \frac{1}{2}\Psi_{(O_2)}\mathcal{M}^{0.7} - s\mathcal{M} - \frac{1}{2}\Omega_{(O_2)}[\beta(N+r) - w\mathcal{C}]\,, \tag{1}$$

$$\frac{d\mathcal{O}}{dt} = \Omega_{(O_2)}N - (1-\Omega_{(O_2)})r - \Psi_{(O_2)}\mathcal{M}^{0.7} - s\mathcal{M} - (1-\Omega_{(O_2)})[\beta(N+r) - w\mathcal{C}]\,, \tag{2}$$

and

$$\frac{d\mathcal{C}}{dt} = \beta(N+r) - w\mathcal{C}\,. \tag{3}$$

This set of differential equations is coupled and nonlinear because of the term $\mathcal{M}^{0.7}$ as well as the functions $\Omega_{(O_2)}$

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[1] Yoyo atmosphere is a term introduced by Ohmoto et al. (2006) [16] to refer to hypothetical oxygen in Earth's atmosphere prior to the Great Oxidation. However, in the following this term is used as done by Cuntz et al. (2009) [17] to describe possible significant ups and downs of early Earth's atmospheric oxygen amounts, which could also have occurred during or after the Great Oxidation.