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Complex waves in a dielectric waveguide

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HIGHLIGHTS

- Existence of complex waves in a dielectric waveguide of circular cross section is rigorously proved.
- A method for calculating complex waves using a dispersion equation is proposed.
- The location of the wave complex propagation constants is determined.

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ABSTRACT

Existence of two families of symmetric complex waves in a dielectric waveguide of circular cross section is proved. Eigenvalues of the associated Sturm–Liouville problem on the half-line are determined.

1. Introduction

A dielectric waveguide of circular cross section, a dielectric rod (DR), is a basic structure in the broad family of open waveguides. Many works (e.g. [1–3]) investigate complex waves in open dielectric waveguides. However, *rigorous* proof of their existence remained unsolved. In this work, we report on the proof of the existence of two families of 'surface' and 'pure' symmetric complex waves in DR using the methods set forth by Shestopalov [4] and Shestopalov et al. [5].

2. Statements and results

Determination of symmetric (azimuthally-independent) TE waves in a DR of radius a described (in Cartesian coordinates x_1, x_2, x_3) via nontrivial solutions to homogeneous Maxwell's equations having the nonzero field components written in

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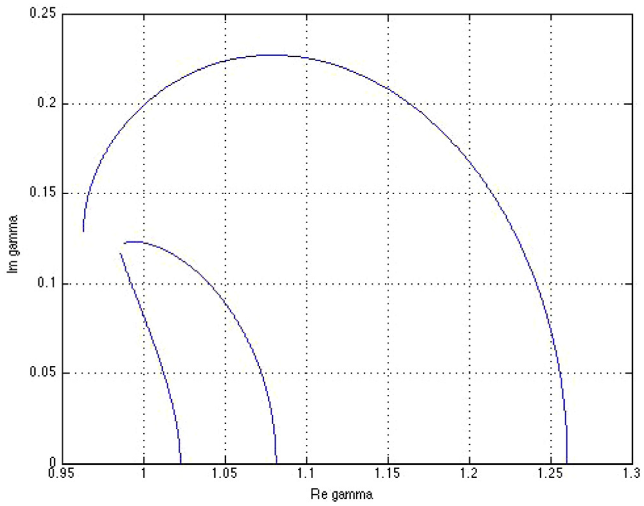

Fig. 1. Dynamics of $\gamma_1(\Im\epsilon)$ on the complex γ-plane at $\kappa = 1$ and $\Re\epsilon = 8, 10, 13$ (left, central and right curves)as $t = \Im\epsilon$ increases from 0 to 7.34.

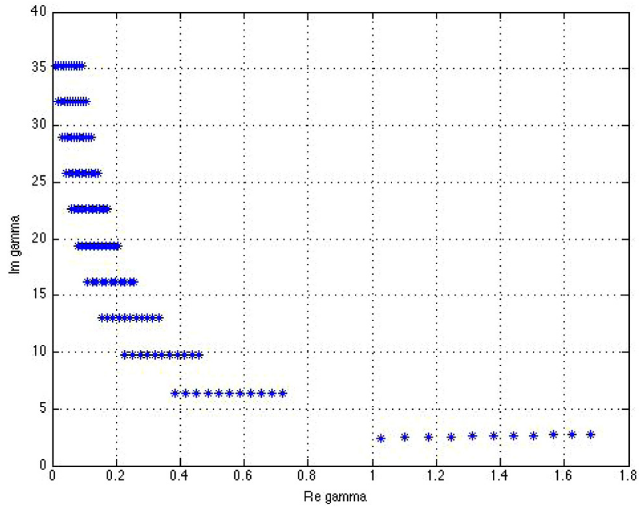

Fig. 2. The first 11 zeros of $\tilde{F}_D$ on the complex γ-plane at 11 different values of $\epsilon = 10 + (10 - 0.2(k-1))i$, $k = 1, \ldots, 11$.

polar coordinates with $r = \sqrt{x_1^2 + x_2^2}$ as

$$\mathbf{H} = [0, H_2(r, x_3), 0], \quad \mathbf{E} = [E_1(r, z), 0, E_3(r, x_3)], \tag{1}$$

$$E_1 = -\frac{i\beta}{k_s^2}\frac{d\phi}{dr}e^{-i\beta x_3}, \quad E_3 = \phi(r)e^{-i\beta x_3}, \quad H_2 = -\frac{i\omega\epsilon}{k_s^2}\frac{d\phi}{dr}e^{-i\beta z}, \tag{2}$$

is reduced to a singular Sturm–Liouville problem on the half-line

$$\mathcal{L}\phi \equiv \frac{1}{r}\frac{d}{dr}(r\frac{d\phi}{dr}) + k_s^2\phi = 0,\ r > 0,\ \ [\phi]|_{r=a} = \left[\frac{\epsilon}{k_s^2}\frac{d\phi}{dr}\right]\Big|_{r=a} = 0, \tag{3}$$

$$\phi \in C[0, +\infty) \cap C^2(0, a) \cap C^2(a, +\infty),\ k_s^2 = \begin{cases} k_0^2 - \beta^2,\ r > a, \\ \epsilon k_0^2 - \beta^2,\ r < a; \end{cases} \tag{4}$$

$$\phi(r) = AH_0^{(1)}(r\tilde{w}),\ r > a,\ \tilde{w} = \frac{i}{a}\sqrt{u^2 - x^2} = k_0\sqrt{1-\gamma^2}. \tag{5}$$