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Time-evolution of age-dependent mortality patterns in mathematical model of heterogeneous human population

Demetris Avraam [a], Séverine Arnold (-Gaille) [b], Dyfan Jones [a], Bakhtier Vasiev [a,*]

[a] Department of Mathematical Sciences, University of Liverpool, Liverpool, UK
[b] Department of Actuarial Science, Faculty of Business and Economics (HEC Lausanne), University of Lausanne, Lausanne, Switzerland

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ABSTRACT

The widely-known Gompertz law of mortality states the exponential increase of mortality with age in human populations. Such an exponential increase is observed at the adulthood span, roughly after the reproductive period, while mortality data at young and extremely old ages deviate from it. The heterogeneity of human populations, i.e. the existence of subpopulations with different mortality dynamics, is a useful consideration that can explain age-dependent mortality patterns across the whole life-course. A simple mathematical model combining the heterogeneity of populations with an assumption that the mortality in each subpopulation grows exponentially with age has been proven to be capable of reproducing the entire mortality pattern in a human population including the observed peculiarities at early- and late-life intervals. In this work we fit this model to actual (Swedish) mortality data for consecutive periods and consequently describe the evolution of mortality dynamics in terms of the evolution of the model parameters over time. We have found that the evolution of the model parameters validates the applicability of the compensation law of mortality to each subpopulation separately. Furthermore, our study has indicated that the population structure changes so that the population tends to become more homogeneous over time. Finally, our analysis of the decrease of the overall mortality in a population over time has shown that this decrease is mainly due to a change in the population structure and to a lesser extent to a reduction of mortality in each of the subpopulations, the latter being represented by an alteration of the parameters that outline the exponential dynamics.

1. Introduction

Mathematical modelling of biological processes such as longevity, ageing and mortality is of interest for many scientists working on various subjects including demography, biology, statistics and actuarial sciences. The event of death and the forces that cause it have puzzled and inspired many philosophers and scientists from the 17th century onwards. Great works such as those by Joseph Addison (1672–1719), Karl Pearson (1857–1936) and Benjamin Gompertz (1779–1865) give us insights on the development of the concept of mortality over the past few centuries (see (Turner and Hanley, 2010) for a review). Addison in his allegorical essay "The vision of Mirza" (Addison, 1711) imagined the human life as a walkthrough over a bridge, "the bridge of human life", where hidden pitfalls open periodically and the people above them fall down and disappear, the forces causing death being then external. Almost two centuries after Addison, Pearson considered death as a random event and decomposed the entire mortality curve into five different phases, described by five different probability distributions (Pearson, 1897). Pearson's concept can be represented with humans crossing the bridge of life, where at each one of the five stages, a marksman attempts to kill them. From one stage to the next the precision of the marksman's weapon improves (five different precisions for the five different age groups) and consequently the chance of death increases. On the other hand, the work by Gompertz (1825) is of greater importance as he was the first who considered death to be caused by internal forces in organisms and proposed a model for the force of mortality. According to Gompertz, the mortality force increases in a geometrical progression within a wide age-range of lifespan, that is from sexual maturity to considerably old ages. This conception is confirmed by many observations and is known as the Gompertz law of mortality. Mathematically, the Gompertz law represents the mortality rate m_x at age x, as an exponential function of age

$$m_x = m_0 e^{\beta x} \tag{1}$$

where m_0 is the initial mortality (can be considered as mortality rate at age 0) and β is the mortality coefficient that gives the rate of change of mortality with age (strictly speaking the age x in the Gompertz law can only be varied in a certain range, i.e. between 20 and 80 years).

Observed mortality data and the theoretical force of mortality are generally plotted in a semi-logarithmic scale where the exponential

* Corresponding author.
E-mail address: bnvasiev@liv.ac.uk (B. Vasiev).

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increase of mortality (Eq. (1)) is represented by a straight line. Graphically the actual mortality data generates patterns which have certain common features as well as some quantitative differences as compared to different cohorts and periods. A typical mortality pattern (Fig. 1) originates from the initial mortality at age zero, falls down to a minimum point (approximately at the age of 10), increases to a local maximum (around the age of 25), then slightly decreases or remains constant and after the age of 35–40 advances exponentially satisfying the Gompertz law. At extreme old ages (above the age of 100) there is no common evidence on how the mortality curve behaves as the reported observations are controversial and provided with different explanations (Gavrilov and Gavrilova, 2011; Greenwood and Irwin, 1939; Olshansky, 1998). Various statements made about the mortality dynamics at old ages include the mortality levelling-off or so-called "late-life mortality plateau" (Curtsinger et al., 2006; Economos, 1979; Mueller and Rose, 1996), the late-life mortality deceleration (Depoid, 1973; Gavrilov and Gavrilova, 2001; Horiuchi and Wilmoth, 1998; Thatcher et al., 1998), the decline (Kannisto et al., 1994; Vaupel et al., 1998) or fluctuations at advanced ages (Avraam et al., 2013).

The high initial level of mortality is due to the fact that new-borns are not particularly fit for the new environment they are born into and therefore, a relatively high proportion of them are not able to survive. As the forces of mortality due to environmental factors decrease, death rates decline. Mortality starts then to increase at the age of 10. One can state that mortality should increase exponentially from this age. However in actual mortality data the exponential increase of mortality is observable only after the ages of 35–40 (Fig. 1) as between the ages 10 and 35 it overlaps with a local maximum on the mortality curve. This local maximum is apparent at the reproductive period of lifespan and is commonly called "the accidental hump" as it is related to the external causes of deaths (mainly accidents and maternal deaths) due to the risky behaviour of young adults.

Many studies have focused on the analysis of exponential increase of mortality in the range of ages 30 and above. By comparing parameters describing the exponential dynamics for data taken for different human societies it was found that in developed countries initial mortality, m_0, is lower while the mortality coefficient, β, is higher than these parameters describing data for less developed countries. This phenomenon, namely the inverse relationship between initial mortality and mortality coefficient appears to be fundamental (confirmed by all available data) and called the "compensation law" or "compensation effect" (Gavrilov and Gavrilova, 1979, 1991).

A number of mathematical models have been proposed to analyse mortality dynamics and explain its deviations from the exponential law at early and late life intervals. Some models postulate that a few different processes take place in the population and affect its mortality dynamics (Heligman and Pollard, 1980; Thiele, 1872), while others analyse the impact of population heterogeneity on the dynamics of mortality (Vaupel and Yashin, 1985; Vaupel et al., 1979). A model based on the assumption that the mortality dynamics is indeed underlined by an exponential law and deviations from this law are due to the heterogeneity of human populations has recently been proposed (Avraam et al., 2013). It was shown that the observed age-specific mortality patterns can be reproduced in a model of heterogeneous population consisting of a few (up to four) subpopulations each following the exponential law over all ages.

Time evolution of mortality dynamics in human populations is of great scientific interest and has practical implementations especially for actuaries, who use extrapolation methods to project mortality trends in order to estimate future life expectancy (Booth and Tickle, 2008; Pitacco, 2004), and to price several longevity products. An example of mathematical study of this evolution can be found in Gaille (2012), where the analysis of the evolution of the parameters of two conventional models (Heligman–Pollard and Lee–Carter) is used to forecast the Swiss mortality rates and to study the impact of longevity on Swiss pension funds. Mathematical analysis of the evolution of mortality dynamics could also be useful for demographers (to derive inferences on the population variance) and for biologists (to understand genetics underlying the evolutionary process of ageing).

In this work we aim to describe the evolution of mortality dynamics as time evolution of the parameters in the model of a heterogeneous population (Avraam et al., 2013) so that we could gain insights in the processes governing mortality reductions over the past century. We introduce our model in Section 2 and used mortality data in Section 3. In Section 4 we fit the model to various mortality data (cut at a certain age or including/excluding the extrinsic death factors) for consecutive periods and analyse the evolution of the model parameters. The results demonstrate that the population's structure is altered through time and a relative homogenization of the population occurs, explaining an important part of mortality reductions during the 20th century. The analysis also indicates that changes in the initial mortality and mortality coefficient of the exponential law for all subpopulations are in line with the compensation law. Discussion of presented results is provided in Section 5.

2. Mathematical model and fitting procedure

In this work we use a previously proposed model (Avraam et al., 2013) where a human population is considered as heterogeneous and composed of a number of subpopulations. The subpopulations are assumed to obey an exponential law, as given by Eq. (1), but differ in their mortality parameters (initial mortality, m_0, and mortality coefficient, β). The mortality of the entire population is modelled as a mixture of weighted exponential terms. The weights represent the relative sizes

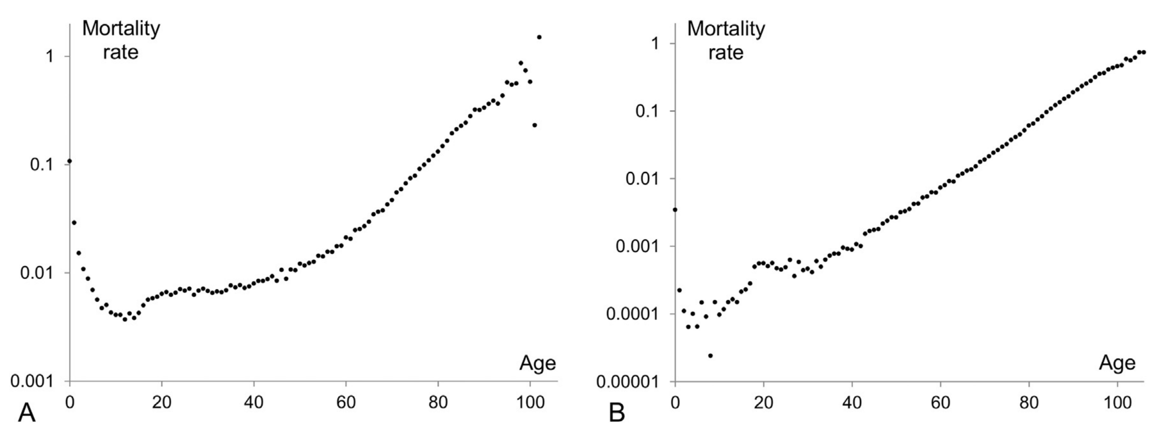

Fig. 1. Mortality rates for the Swedish population in the period 1900 (panel A) and 2000 (panel B) presented in a semi-logarithmic scale. The data are taken from the Human Mortality Database, http://www.mortality.org.