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A heuristic model selection scheme for representing hot flow data using the hot torsion test results

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ABSTRACT

The problem of "model selection" for expressing a wide range of constitutive behaviour adequately using hot torsion test data was considered here using a heuristic approach.

A model library including several nested parametric linear and non-linear models was considered and applied to a set of hot torsion test data for API-X 70 micro-alloyed steel with a range of strain rates and temperatures. A cost function comprising the modelled hot strength data and that of the measured data were utilized in a heuristic model selection scheme to identify the optimum models. It was shown that a non-linear rational model including ten parameters is an optimum model that can accurately express the multiple regimes of hardening and softening for the entire range of the experiment. The parameters for the optimum model were estimated and used for determining variations of hot strength of the samples with deformation.

1. Introduction

Accurate hot strength data are required to design or optimise an industrial hot forming process. The models generated from such data, also known as constitutive models, are an essential component for numerical simulations of the hot forming processes. Such models are especially useful in conjunction with finite element analysis in which the model can be easily incorporated and used to design and optimise processes and product. Parametric representation of the hot strength data is also a useful alternative for the commonly used constitutive models and could be integrated in mathematical models for the kinetics of (SRX) and dynamic recrystallization (DRX).

The constitutive behaviour of materials during hot working is quite complex in nature. It is usually described by either phenomenological or empirical/semi-empirical methods. Alternatively, artificial neural network models can be used to model hot flow stress of materials.

Phenomenological constitutive equations such as that of Estrin and Mecking [1] are useful for understanding the underlying theory behind hot deformation. However they need further development before they can be applied in industry.

Many empirical/semi-empirical models have been introduced to represent hot flow behaviour of the materials. Such models usually have a very limited scope of application. More detailed models of this type describe time related transformations such as fraction of transformed microstructure, final texture and mechanical properties. These models use quantitative relationships between the microstructural and kinetic parameters and the process variables, i.e. strain, strain rate, temperature and time (for example see [2]). Typical example of empirical models is the work by Rao and Hawbolt [3] and Lin et al. [4] in which they have defined their primary parameters appearing in a Zener–Hollomon type constitutive model that are in turn functions of strain using secondary parameters. For both models, identification of 9 and 25 parameters, respectively, are needed to evaluate flow stress from the proposed constitutive model. However, the predictions made by the models are not very accurate and even the predictions become worse when extrapolation is attempted. Kim et al. [5] indicated the shortcomings of using some existing constitutive models. They showed that the use of these models in conjunction with the results of compression test and torsion tests could lead to different estimations of hot flow data. Such errors could partially be reduced by using a better model for the constitutive behaviour.

Artificial Neural Network (ANN) and Integrated Phenomenological ANN techniques, used by Kong et al. [6], Narayan et al. [7], Mandal et al. [8] and Reddy et al. [9], are fundamentally different in their approach to constitutive modelling and material processing control. The range of input in these models varies from the product of strain and stress, work hardening coefficient, chemical composition, temperature, strain rate and initial microstructure. The models often use hot torsion data (Hodgson et al. [10]) or compression to train their network. The output is usually the flow

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stress. These techniques, however, require a comprehensive database and rule sets for training the network for enabling the resulting model to approximate all of the laws of mechanics that the actual material or process obeys.

On the numerical front, the inverse technique has been used to identify the parameters of a constitutive model. Khoddam et al. [11] developed a rigid viscoplastic FE code for parameter estimation of power law and hyperbolic form constitutive equations. Gavrus et al. [12] developed a similar FEM code that could identify the rheological parameters corresponding to a generalized viscoplatic Norton–Hoff constitutive equation from the results of the hot torsion test.

Although a large number of researchers in constitutive modelling have used different techniques to estimate the parameters in each of above mentioned models, their parameter estimation techniques are bounded by a pre-assumed model. Systematic selection of the best model for a given material, i.e. the *model selection problem*, has not been investigated sufficiently in the existing literature. The aim of this investigation was to apply a heuristic model selection approach to identify an adequate mathematical representation of hot flow behaviour using the hot torsion test data. In order to substantiate the approach, the hot torsion test results for an API X70 micro-alloyed steel were used.

2. Heuristic model selection

In statistics literature, the problem of finding the most appropriate and concise model to express given data is called the "model selection problem" [13]. The solution is a statistical model from a set of potential models, given data. One needs to determine the principle behind a series of observations which is often linked directly to a mathematical model predicting those observations. Such a "mathematical model" will be abbreviated in this article as "model".

Once the set of possible models are decided, an analysis is needed to select the optimum model. The model selection technique will balance goodness and complexity of the model. Goodness of the model is generally determined using an approximation of likelihood ratio, leading to a Chi-squared test. Goodness of fit and the Chi-squared test are defined for the case of hot flow data later in this article. The complexity is generally related to the number of parameters in the model.

Model selection for the hot torque-twist data were carried out in this work to find the optimum flow stress model.

2.1. Model selection criteria

A combination of visual inspection, use of a quantified factor and some background considerations were used in this work to compare the goodness of each model and to exclude inadequate models from the model library, respectively, and eventually to select the optimum model.

2.1.1. The goodness of the fit

Given a set of torque-twist data, if data point (M_i, θ_i) has a modelled value of f_i and an average value of $\overline{M}$, respectively, for quantitative comparison of the models the coefficient of determination (otherwise known as the R – squared value), may be considered in which:

$$R^2 = 1 - \frac{\sum_i (M_i - f_i)^2}{\sum_i (M_i - \overline{M})^2} \tag{1}$$

The closer this value is to 1, the more accurate the model is deemed to be to the data.

2.1.2. Background knowledge

The existence of noise in the hot torsion test data, which is usually of high frequency, is inevitable due to un-wanted deformation rate changes during deformation at elevated temperature (issues related to speed control in the test rig), backlashes and clearances in torque measuring mechanism/chain, vibration of structure and finally a low ratio of signal to noise. The model should be able to distinguish between two types of fluctuations in a flow curve including the change of hardening behaviour and noise in torque – twist data.

In addition to the above criterion, the following general considerations should be kept in mind:

- *Complexity*: a bad model may fit the data poorly or needs too many terms.
- *Smoothness*: minimum noise type fluctuations are desirable.
- *Completeness*: the model should have the potential for capturing all types of high level information such as hardening, softening and an asymptote end, when a perfectly plastic behaviour is expected, or a combination of them over the entire range of deformation.
- *Accuracy*: the model has to present the accurate location of the peak stress which is used to predict the onset of dynamic recrystalization.
- *Universality*: the model must be capable of providing universally adequate fits for different shapes of data obtained from the hot torsion test.

The second type of background information is related to the competing effects of active hardening and softening mechanisms during hot deformation of the material. In absence of any softening mechanism during hot deformation, the material exhibits monotonic work hardening behaviour during plastic deformation. In certain conditions, dynamic recovery (DRV) results typically in monotonic hardening to a steady state plateau, where an equilibrium is reached between dislocation generation and annihilation. During DRX, dislocations are eliminated through replacement of deformed grains by new grains. This is a typical case for recrystalization of austenitic steel during hot deformation. It generally consists of a work-hardening peak followed by softening to a steady-state level.

2.2. Data modelling and model selection

Representing hot strength data by a mathematical model is not achieved by finding an expression which passes through all measured data. This is due to the fact that the number of existing data is much more than the required parameters in the model. Also, a model should exclude the error component of the measured data from the real value of the data. It can be assumed the real components of the measured data present a datum and the errors are symmetrically distributed around them. The adjustment process of parameters for a flow stress model is a problem in minimization in many dimensions.

A rather more difficult problem is how to be sure that there is not a much better model in some corner of the model space. In an attempt to solve this problem a model library was proposed. Subsequently, for each model in the library, data modelling was performed. This was followed by a model selection scheme described above.

2.3. Merit function for modelling of the hot torsion test data

Given a set of torque-twist data from the test, if data point (M_i, θ_i) has its own standard deviation σ_i, then the vector of model