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Phase sensitivity evaluation and its application to phase shifting interferometry

Shichao Chen, Yizheng Zhu *

The Bradley Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA 24061, USA

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ABSTRACT

In quantitative phase imaging, sensitivity is a key measure of system reproducibility. Despite continuous experimental breakthroughs in achieving highly sensitive detection, in-depth studies of theoretical constraints on sensitivity are inadequate and comparisons between different techniques are difficult. In this paper, we introduce the method to evaluate the sensitivity of phase shifting interferometry which is a major category of quantitative phase imaging techniques. The method discusses in detail several key concepts of sensitivity evaluation, including a general three-level evaluation framework, a complete interference signal model, Cramér-Rao bound and algorithm sensitivity, algorithm and system efficiencies, as well as energy efficiency of an algorithm. In discussions of specific phase shifting algorithms, we focus on the shot noise-limited model. This simplified model not only reflects the rapid developments in modern detectors that are often dominated by shot noise, but also permits the calculation of theoretical sensitivities based on measured data, which is important in evaluating experimental performance. As examples, we study several common phase shifting interferometric techniques. The results of different techniques are compared to provide insights into algorithm optimization and energy efficiency of sensitivity. A normalized algorithm sensitivity table is also provided for readers to conveniently estimate their system's algorithm sensitivity and compare against experiments.

1. Introduction

Phase shifting interferometry (PSI) has been a well-established category of techniques in the field of optical metrology, sensing and quantitative phase imaging [1–5]. Despite a variety of implementations, it is essentially an on-axis interferometric measurement that generates phase shifts while recording interferograms. By analyzing these interferograms, which are the coherent additions of the optical fields from the test arm and reference arm of an interferometer, optical pathlength (OPL) difference between them can be obtained and can be further interpreted for various applications.

The accuracy of the retrieved OPL value is affected by systematic errors, which are consistent throughout a series of measurements. Its precision, on the other hand, is affected by non-repeatable fluctuations, whose standard deviation is typically defined as measurement sensitivity. This fluctuation is a result of various noise sources, including detector noises, light source fluctuation, environmental vibration and the stochastic behavior of phase/wavelength shifting devices. Temporal sensitivity characterizes the temporal non-repeatability at a fixed location and spatial sensitivity characterizes the spatial non-uniformity within a region. In PSI, modulation and demodulation are mainly performed on a pixel basis, hence temporal sensitivity is of greater interest. Unless otherwise specified, sensitivity in this paper stands for the temporal sensitivity. A smaller value means the system can perform more stable measurements and is therefore more sensitive to minute OPL changes in the sample.

Sensitivity evaluation allows one to understand and quantify how well a system performs in terms of its measurement capability. Does it provide the best accuracy it is capable of? How much, if any, of the experimental sensitivity is contributed by environmental or hardware instabilities? How can one improve the system and/or algorithm to achieve the same sensitivity with lower light exposure of the specimen? This last question is of particular interest to live cell imaging, where efforts to minimize light damage are always appreciated. Sensitivity measured from actual acquisitions is the familiar experimental sensitivity (EXP). More importantly, in order to conduct a complete sensitivity evaluation, we should also consider the corresponding theoretical constraints as well as algorithm and system efficiencies. These concepts will be discussed in detail in the next section. Analyses of theoretical sensitivities require proper statistical modelling of the noise behaviours in

* Corresponding author.
E-mail address: yizhu1@vt.edu (Y. Zhu).

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the signals. The earliest works, such as those studying Cramér-Rao bound sensitivity (CRB) as the fundamental sensitivity limit [6–9], and those studying algorithm sensitivity (ALG) [10–12], all assume a uniform Gaussian noise distribution. However, advances in modern camera and detector technology have been pushing their operation into the shot noise-limited regime, which differs substantially from uniform Gaussian noise. Since shot noise depends directly on signal intensity, it permits the characterization of noise entirely from measured data. Therefore, it calls for a reexamination of the sensitivity theory in optical interferometry.

Our recent work has demonstrated a three-level sensitivity evaluation framework using a wavelength shifting interferometry (WSI) technique as an example [13]. In that paper, EXP has shown excellent agreement with the shot noise-limited ALG calculated from measured interference intensity. Compared to WSI, PSI has substantially more variations in implementations. Different number and value of phase steps lead to different modulation techniques. Even for the same modulation, there can exist various demodulation algorithms. Comparisons are thus needed not only vertically to examine algorithm efficiency and system efficiency, but also horizontally to compare different techniques, algorithms, and systems. In this paper, we introduce a general framework of sensitivity evaluation and its key concepts. We then focus on the shot noise-limited CRB and ALG for PSI techniques and propose an energy normalization scheme for their sensitivities to enable a fair comparison. Meanwhile, the shot noise-limited model is shown to lead to discrepancies between CRB and ALG for some techniques, which have not been observed with Gaussian noise model. These findings suggest possibilities of algorithm improvements.

The paper is outlined as follows. In Section 2, general formulae of CRB and ALG, and in particular their shot noise-limited case, will be derived using the general form of PSI signals. We will then analyze and compare several common techniques and algorithms in Section 3. A convenient normalized ALG lookup table will be provided for readers to estimate their system's potential ALG and compare against EXP. Finally, we will summarize our results and their implications.

2. Method

Proper modelling of signal and noise statistics is the prerequisite to perform sensitivity evaluation. We will start with a complete signal model, discuss its Gaussian and Poisson approximations, and then focus on its shot noise limited form. The methods to evaluate all three level sensitivities and related indices are introduced subsequently.

2.1. The complete signal model

The noise-free (average) PSI intensity at each phase step can be written as:

$$\overline{I_n} = \alpha[1 + V\cos(k_0L + \varphi_n)] = \alpha + \beta\cos(k_0L + \varphi_n), n = 1 \sim N, \quad (1)$$

where the DC term, α, represents the combined intensity of the sample and reference arms, and β is the amplitude of the interference term with k_0L (k_0: wavenumber; L: OPL) as the initial phase and $\mathbf{\Phi} = [\varphi_1, \varphi_2, \dots \varphi_n]$ as the additional phase shift for the n frames. The intermediate step containing the visibility $V = \beta/\alpha$ (also known as normalized contrast or modulation depth) is another typical representation. For the ease of derivation in this section, we will use the last representation with an unknown parameter set of $\mathbf{\Omega} = [\omega_1, \omega_2, \omega_3] = [\alpha, \beta, L]$.

The noise-free $\overline{I_n}$ is the ideal case. For actual acquisitions, we use I_n to specify the measured, noise-corrupted intensity readout. In cameras or photo-detectors, this direct readout is a value in analog-to-digital unit (ADU). The actual number of photo-electrons $\mathbf{x} = [x_1, \dots, x_N]$ collected by the detector can be expressed as $x_n = gI_n$, where g is the camera gain. In general, we can express the n-th observation as

$$x_n = s_n + d_n + r_n. \quad (2)$$

In this expression s_n is the number of photo-generated electrons representing the interference signal with shot noise. It follows Poisson distribution with mean value of $g\bar{I}_n$ and expressed as $Po(s_n; g\bar{I}_n)$. Dark noise d_n, representing the number of thermally generated electrons (dark current), and readout noise r_n, representing readout electron fluctuations, are also two common well understood noises. We assume uniform dark current and readout noise for all sampling points. Hence d_n follows Poisson distribution $Po(d_n; g\bar{I}_d)$ with $\bar{I}_d$ being the dark current intensity in ADU, and the readout noise r_n is modeled as a zero-mean Gaussian distribution with σ_r^2 as its variance, expressed as $N(r_n; 0, \sigma_r^2)$ [14,15].

2.2. Gaussian and Poisson approximations

The complete model is a sum of Gaussian and Poisson random variables, presenting difficulties in further derivation. Two approaches may be used to approximate the model. The first one approximates s_n and d_n as Gaussian variables, so that the sum x_n follows a Gaussian distribution. Alternatively, we can approximate Gaussian RV r_n as a Poisson distribution so that x_n, sum of all Poisson random variables, can still be treated as Poisson.

In the Gaussian approximation approach, the approximations can be made on s_n and d_n to convert the whole model to be entirely Gaussian. Here we use a Gaussian distribution $N(x; \mu, \mu)$ to approximate a Poisson distribution $Po(x; \mu)$. Therefore, s_n and d_n can be approximated as Gaussian random variables following $N(s_n; g\bar{I}_n, g\bar{I}_n)$ and $N(d_n; g\bar{I}_d, g\bar{I}_d)$ respectively. In most cases, when $g\bar{I}_n$ and $g\bar{I}_d$ are not too small, this type of approximation is highly accurate. With all three components being Gaussian, x_n thus becomes a Gaussian random variable as well. The complete interference signal model follows $N(x_n; \mu_n - \sigma_r^2, \mu_n)$ with a mean value of $\mu_n = g\bar{I}_n + g\bar{I}_d + \sigma_r^2$.

The Poisson approximation, on the other hand, will convert the only exception, r_n, which has a Gaussian distribution $N(r_n; 0, \sigma_r^2)$, to Poisson distribution. Practically, a Gaussian distribution $N(x; \mu, \mu)$ can be very well approximated as a Poisson distribution $Po(x; \mu)$ when μ is not too small. Therefore, a shifted read noise $r_n + \sigma_r^2$, which follows $N(r_n; \sigma_r^2, \sigma_r^2)$, can be considered to be $Po(r_n; \sigma_r^2)$. Adding σ_r^2 to both sides of Eq. (2) leads to a shifted complete model,

$$x'_n = x_n + \sigma_r^2 = s_n + d_n + (r_n + \sigma_r^2), \quad (3)$$

which becomes the sum of three independent Poisson random variables. The shifted complete model now follows Poisson distribution $Po(x'_n; \mu_n)$, with its rate being the combined rate of all three components, i.e. $\mu_n = g\bar{I}_n + g\bar{I}_d + \sigma_r^2$.

2.3. Shot noise-limited model

In the complete model or either one of its approximations listed above, shot noise component is directly determined by the noise-free interference intensity but dark noise and read noise of the detectors require additional experimental characterizations. Fortunately, with the advances in modern photo-detectors, shot noise has almost always been the dominated term and hence we can neglect the two additional terms in the complete model.

The shot noise-limited model is consistent with the Poisson approximation except for saving the need of a shifted signal. Each

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