Stability and bifurcation analysis in a predator–prey system with Michaelis–Menten type predator harvesting

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A B S T R A C T

The stability and bifurcation analysis for a predator–prey system with the nonlinear Michaelis–Menten type predator harvesting are taken into account. The existence and stability of possible equilibria are investigated. Specially, the stability of some positive equilibria is determined by using numerical simulation method due to the fact that the expressions of determinant and trace of the Jacobian matrix at these equilibria are very complex. The rigorous mathematical proofs of the existence of saddle–node bifurcation and transcritical bifurcation are derived with the help of Sotomayor's theorem. Furthermore, in order to determine the stability of limit cycle of Hopf bifurcation, the first Lyapunov number is calculated and a numerical example is given to illustrate graphically. Choosing two parameters of the system as bifurcation parameters, we prove that the system exhibits Bogdanov–Takens bifurcation of codimension 2 by calculating a universal unfolding near the cusp. Numerical simulations are carried out to demonstrate the validity of theoretical results. Our research will be useful for understanding the dynamic complexity of ecosystems or physical systems when there is the nonlinear Michaelis–Menten type harvesting effect on predator population. This kind of nonlinear harvesting is more realistic and reasonable than the model with constant-yield harvesting and constant-effort harvesting. It can be thought as a supplement to existing literature on the dynamics of this system, since there is little literature involved in nonlinear type harvesting for the system up to now.

1. Introduction

Ever since the pioneering work of Lotka and Volterra who first proposed two differential equations that describe the relationship between predators and prey in 1925 and 1926, respectively [1], predators and prey models have been continuously researched over the last one hundred years due to its significance in many problems [2–4]. From the point of view of human needs, the exploitation of biological resources, the management of renewable resources, and the harvesting of populations are commonly human purpose

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of achieving the economic interest in fishery, forestry, and wildlife management [5,6]. Hence, this is the motivation to introduce and to consider the harvesting of populations in predator–prey models. Predator–prey models with harvesting and the role of harvesting in the management of renewable resources are studied extensively by many authors [7–12]. In 1979, May et al. [13] have proposed two types of harvesting regimes: (i) constant-yield harvesting, which is described as harvested biomass independent of the size of the population, and (ii) constant-effort harvesting, i.e., proportional harvesting, which is described as harvested biomass proportional to the size of the population.

In terms of predator–prey systems with constant-yield harvesting, Huang et al. [14] systematically studied the dynamical properties of a predator–prey model of Holling and Leslie type with nonzero constant-yield prey harvesting. They have shown that the harvested model can exhibit richer dynamics compared to the model with no harvesting, such as appearance of numerous kinds of bifurcations for the model, including saddle–node bifurcation, Hopf bifurcation, repelling and attracting Bogdanov–Takens bifurcations of codimensions 2 and 3. Sen et al. [7] focused on the global dynamics of a predator–prey system when predator is provided with additional food as well as harvested at a constant rate. Refs. [15,16] also have paid great attention to study the effect of constant-yield harvesting in predator–prey models.

In terms of constant-effort harvesting, a ratio-dependent predator–prey model in which the prey is continuously being harvested at a linear function rate was studied by Xiao et al. [17]. They proved that the system has different behaviors for various parameter values. Particularly, there exist areas of coexistence in which both populations become extinct, and areas of "conditional coexistence" depending on the initial values. Makinde [18] developed an algorithm to approach the solution of the ratio-dependent predator–prey system with constant effort harvesting. In addition to these work mentioned above, the effect of constant-effort harvesting in predator–prey models has been studied in [19,20], and the reference therein as well.

However, it is well-known that nonlinear type harvesting is more realistic from biological and economic points of view [21] and is better than the constant-yield harvesting and constant-effort harvesting [22,23]. There are two main reasons. On the one hand, harvesting does not always occur with constant yield or constant effort [6]. On the other hand, constant-effort harvesting embodies several unrealistic features and limitations. Traditionally, the constant effort catch-rate function is taken into account in the form $h(E, x) = qEx$ based on the catch-per-unit-effort hypothesis, where E denotes effort and c is a constant. We can see that h tends to infinity as the effort E tends to infinity if the population x is finite and fixed, or as the population x tends to infinity if the effort E is finite and fixed [24]. The harvesting term $h(E, x) = \frac{qEx}{cE+lx}$ proposed firstly by Clark [25] is the so-called Michaelis–Menten type functional form of catch rate, where q is the catchability coefficient, E is the external effort devoted to harvesting, c and l are constants. Now we have $\lim_{E\to\infty} h(E, x) = \frac{q}{c}x$, and $\lim_{x\to\infty} h(E, x) = \frac{q}{l}E$ [26,27]. Hence, these restrictive features which we have mentioned-above are largely removed. For more details about this kind of harvesting type one can see Ref. [27]. A modified Leslie–Gower predator–prey model with time delay and the Michaelis–Menten type prey harvesting was investigated by Yuan et al. [28]. They obtained the critical conditions for the saddle–node-Hopf bifurcation, and gave the completion bifurcation set by calculating a universal unfolding near the saddle–node-Hopf bifurcation point. In [29], Zhang et al. discussed a reaction–diffusion predator–prey model with non-local delay and Michaelis–Menten-type prey-harvesting. They revealed that the discrete and non-local delays are responsible for a stability switch in the model system, and a Hopf bifurcation occurs as the delays pass through a critical value.

May et al. [13] proposed the following model to describe the interaction of predators and their prey subjected to various harvesting regimes:

$$\begin{cases} \dot{x} = r_1 x \left(1 - \dfrac{x}{K}\right) - axy - H_1, \\ \dot{y} = r_2 y \left(1 - \dfrac{y}{bx}\right) - H_2, \end{cases} \tag{1}$$