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Well-posedness and persistence properties for two-component higher order Camassa–Holm systems with fractional inertia operator[☆]

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ABSTRACT

In this paper, we study the Cauchy problem for a two-component higher order Camassa–Holm systems with fractional inertia operator $A = (1-\partial_x^2)^r, r \geq 1$, which was proposed by Escher and Lyons (2015). By the transport equation theory and Littlewood–Paley decomposition, we confirm the local well-posedness of solutions for the system in nonhomogeneous Besov spaces $B^s_{p,q} \times B^{s-2r+1}_{p,q}$ with $1 \leq p, q \leq +\infty$ and the Besov index $s > \max\left\{2r + \frac{1}{p}, 2r + 1 - \frac{1}{p}\right\}$. Moreover, we demonstrate the local well-posedness in the critical Besov space $B^{2r+\frac{1}{2}}_{2,1} \times B^{\frac{3}{2}}_{2,1}$. On the other hand, the propagation behavior of compactly supported solutions is examined, namely whether solutions which are initially compactly supported will retain this property throughout their time of evolution. Finally, we also establish the persistence properties of the solutions to the two-component Camassa–Holm equation with $r = 1$ in weighted $L^p_\phi := L^p(\mathbb{R}, \phi^p(x)dx)$ spaces for a large class of moderate weights.

1. Introduction

In this paper, we consider the following Cauchy problem

$$\begin{cases} m_t = \alpha u_x - b u_x m - u m_x - \kappa \rho \rho_x, & m = Au, \quad\quad t > 0, \quad x \in \mathbb{R}, \\ \rho_t = -u\rho_x - (b-1)u_x\rho, & \alpha_t = 0, \quad\quad t > 0, \quad x \in \mathbb{R}, \\ u(x,0) = u_0(x), & \rho(x,0) = \rho_0(x), \quad t = 0, \quad x \in \mathbb{R}, \end{cases} \tag{1.1}$$

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where the inertia operator $A = (1-\partial_x^2)^r$ belongs to the class of fractional Sobolev norms $r \geq 1$, and the constants $b \in \mathbb{R}, \kappa \in \mathbb{R}$.

Obviously, if $\rho \equiv 0, A = 1-\partial_x^2$ and $\alpha_x = 0$ the Eq. (1.1) becomes a one-component family of equations which are parameterized by b:

$$u_t - u_{xxt} + \alpha u_x + (b+1)uu_x = bu_x u_{xx} + uu_{xxx}. \tag{1.2}$$

This family of so-called b-equations possess a number of structural phenomena which are shared by solutions of the family of equations (c.f.[1–3]). By using Painlevé analysis, there are only two asymptotically integrable equations within this family: the Camassa–Holm (CH) equation (Eq. (1.2) with $b = 2$, c.f. [4,5]) and the Degasperis–Procesi (DP) equation (Eq. (1.2) with $b = 3$, c.f. [6]). Integrable equations have widely been studied because they usually have very good properties including infinitely many conservation laws, infinite higher-order symmetries, bi-Hamiltonian structure, and Lax pair which make them solved by the inverse scattering method. Conserved quantities are very feasible for proving the existence of global solution in time, while a bi-Hamiltonian formulation helps in finding conserved quantities effectively. The advantage of the CH and DP equations in comparison to the KdV equation, lies in the fact that they not only have peaked solitons but also model the peculiar wave breaking phenomena (c.f. [5,7]), and hence they represent the first examples of integrable equations which possess both global solutions and solutions which display wave-breaking in finite time, c.f. [8,9,7].

In recent years, a number of integrable multi-component generalizations of the CH equation have been studied extensively. One of them is the following family of two-component system parameterized by b

$$\begin{cases} m_t = \alpha u_x - bu_x m - um_x - \kappa\rho\rho_x, \quad m = u - u_{xx}, \\ \rho_t = -u\rho_x - (b-1)u_x\rho, \quad \alpha, b \in \mathbb{R}. \end{cases} \tag{1.3}$$

Apparently, the two-component Camassa–Holm system [10], and the two-component Degasperis–Procesi system [11] are included in Eq. (1.3) as two special cases with $b = 2$ and $b = 3$, respectively. Recently, Escher et al. [12] presented the hydrodynamical derivation of the system (1.3), as a model for water waves with α a constant incorporating an underlying vorticity of the flow. They also proved the local well-posedness of (1.3) using a geometrical framework, studied the blow-up scenarios and global strong solutions of (1.3) on the circle.

All of these hydrodynamical models have a geometrical interpretation in terms of a geodesic flow on an appropriate infinite dimensional Lie group. The seminal work of Arnold [13] reformulated the Euler equation describing an ideal fluid, as a geodesic flow on the group of volume preserving diffeomorphisms of the fluid domain. Following this, Ebin and Marsden [14] reinterpreted this group of volume preserving diffeomorphisms as an inverse limit of Hilbert manifolds. This technique has been used in [15,16] for the periodic CH equation. It was extended to (nonmetric) geodesic flows such as the DP equation in [17] and to right-invariant metrics induced by fractional Sobolev norm (non-local inertia operators) in [18]. In 2009, McLachlan and Zhang [19] studied the Cauchy problem for a modified CH equation derived as the Euler–Poincaré differential equation on the Bott–Virasoro group with respect to the H^k metric, i.e.,

$$m_t + 2u_x m + um_x = 0, \quad m = (1-\partial_x^2)^k u, \ \ k \in \mathbb{N}. \tag{1.4}$$

In [20], Coclite, Holden and Karlsen considered higher order Camassa–Holm equations (1.4) describing exponential curves of the manifold of smooth orientation-preserving diffeomorphisms of the unit circle in the plane. Recently, in [21], Escher and Lyons have shown that the system in Eq. (1.1) corresponds to a metric induced geodesic flow on the infinite dimensional Lie group $\mathrm{Diff}^\infty(\mathbb{S}^1) \circledS C^\infty(\mathbb{S}^1) \times \mathbb{R}$, where $\mathrm{Diff}^\infty(\mathbb{S}^1)$ denotes the group of orientation preserving diffeomorphisms of the circle, $C^\infty(\mathbb{S}^1)$ denotes the space of smooth functions on $\mathbb{S}^1$ while $\circledS$ denotes an appropriate semi-direct product between the pair.