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Global classical solutions of the full compressible Navier–Stokes equations with cylindrical or spherical symmetry

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ABSTRACT

In this paper, we consider the full compressible Navier–Stokes equations in $N(N \geq 2)$ space dimension with cylindrical or spherical symmetric initial data. The global existence of strong and classical solutions is established. The analysis is based on some delicate *a priori* estimates which depend on the assumption $\kappa(\theta) = \theta^q$ where $q \geqslant 0$ and $(\rho_0, \theta_0) \in H^2$, $(u_0, v_0, w_0) \in H_0^1 \cap H^2$. Compared with the results in Wen and Zhu (2014) and Qin, Yang, Yao and Zhou (2015), our results relax the restriction $q > 0$, when there is no initial vacuum and include the global existence of classical solutions for both the cylindrical or spherical symmetric cases, respectively. It should point out that we obtain the global classical solutions with the help of weighted H^3 estimates of (u, v, w, θ).

1. Introduction

It is well known that the full compressible Navier–Stokes equations, which describe the motion of viscous, heat-conducting gas can be written in Eulerian coordinates in $\Omega \subset \mathbb{R}^N$ as follows:

$$\begin{cases} \rho_t + \operatorname{div}(\rho \mathbf{u}) = 0 \\ (\rho \mathbf{u})_t + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla P = \operatorname{div}(\mathcal{T}), \\ (\rho E)_t + \operatorname{div}(\rho \mathbf{u} E) + \operatorname{div}(P \mathbf{u}) = \operatorname{div}(\mathcal{T} \mathbf{u}) + \operatorname{div}(\kappa \nabla \theta), \end{cases} \tag{1.1}$$

here $\mathcal{T}$ is the stress tensor given by

$$\mathcal{T} = \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})') + \lambda \operatorname{div} \mathbf{u} I_N,$$

where I_N is an $N \times N$ unit matrix; $\rho = \rho(\mathbf{x}, t), \mathbf{u} = \mathbf{u}(\mathbf{x}, t) = (u_1, \ldots, u_N)(\mathbf{x}, t)$ and $\theta = \theta(\mathbf{x}, t)$ denote the density, velocity and absolute temperature, respectively; $P = P(\rho, \theta), E = e + \frac{1}{2}|\mathbf{u}|^2, e(\rho, \theta)$ and κ

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are pressure, total energy, internal energy and heat conductivity coefficient, respectively. The viscosity coefficients μ and λ satisfy the following physical restrictions:

$$\mu > 0, \quad 2\mu + N\lambda \geqslant 0;$$

P and e satisfy the second principle of thermodynamics:

$$P = \rho^2 \frac{\partial e}{\partial \rho} + \theta \frac{\partial P}{\partial \theta}.$$

The well-posedness theory of the full compressible

Navier–Stokes equations (1.1) has been extensively studied with a lot of references. In the absence of vacuum (i.e. $\inf \rho_0 > 0$), the local existence of classical solutions to three dimensional system (1.1) in Hölder spaces was obtained by Itaya [1] for Cauchy problem and by Tani [2] for initial boundary value problem, respectively. Matsumura and Nishida [3,4] proved the global existence of classical solutions in three space dimension, when the initial perturbation is small. Kazhikhov and Shelukhi [5] (for polytropic perfect gas with $\mu, \lambda, \kappa = const.$) and Kawohl [6] (for real gas with $\kappa = \kappa(\rho, \theta),\ \mu,\ \lambda = const.$) got the global classical solutions to one-dimensional system in Lagrangian coordinates. In [6], the heat conductivity coefficient $\kappa(\rho, \theta)$ satisfies the assumption

$$\kappa_0(1 + \theta^q) \leqslant \kappa(\rho, \theta) \leqslant \kappa_1(1 + \theta^q), \quad q \geqslant 2 + 2r,\ r \in [0, 1],$$

where κ_0, κ_1 are positive constants. For the polytropic ideal gas, Jiang [7] got spherically symmetric classical large solutions in an exterior domain. On the existence, asymptotic behavior of the weak solutions, we can refer to [8–10] for the existence of weak solutions in 1D and for the existence of spherically symmetric weak solutions in $\mathbb{R}^N$ $(N = 2, 3)$, and refer to [11] for the existence of spherically or cylindrically symmetric weak solutions, and [12] for the existence of variational solutions in a bounded domain in $\mathbb{R}^N$ $(N = 2, 3)$. Recently, for the ideal gas, Qin, Yang, Yao and Zhou [13] studied the global well-posedness for large initial data and the vanishing shear viscosity limit with a boundary layer to the compressible Navier–Stokes system with cylindrical symmetry under a general condition on the heat conductivity coefficient, which included the constant case.

In the presence of vacuum (i.e. ρ may vanish), the related results about the well-posedness to (1.1) are few, we can refer to [14] for the global existence of weak solutions in $\mathbb{T}^3$ or $\mathbb{R}^3$ with special pressure, viscosity and heat conductivity, and [15] for the existence of variational solutions in dimension $N \geq 2$. It is worth mentioning that Cho and Kim [16] (for the perfect gas with $\mu,\ \lambda,\ \kappa = const.$) obtained the existence and uniqueness of local strong solutions for $N = 3$. Next, Wen and Zhu [17] got the existence and uniqueness of global strong and classical solutions in one dimension with large initial data, in which, they made the assumption: $\kappa(\theta) = O(1 + \theta^q), q > 2 + 2r,\ r \geqslant 0$. Recently, under the assumption $\kappa(\theta) = O(1 + \theta^q),\ q > r,\ r \geqslant 0$, Wen and Zhu [18] obtained the existence and uniqueness of global cylindrical or spherical symmetric classical solutions.

For compressible isentropic Navier–Stokes equations (i.e. no temperature equation in (1.1)), there are many results about the existence and large time behavior of the global smooth(weak) solutions to compressible Navier–Stokes equations with density dependent viscosity or constant viscosity and with or without vacuum, refer to [19–28] and references cited therein.

In this paper, we will consider the full compressible Navier–Stokes equations (1.1) with cylindrical or spherical symmetric initial data, and prove the global existence of strong and classical solutions without initial vacuum. There are two main theorems in this paper. In Theorem 2.1, we get the global existence of strong solutions. In Theorem 2.2, we get further regularity of solutions for the same regular initial data and get the global classical solutions. Compared with [18], there are two differences. On the one hand, for the heat conductivity coefficient $\kappa(\theta) = \theta^q$ where $q \geqslant 0$, it relaxes the condition $q > r$ $(r \geq 0)$ in [18]; on the other hand, in order to avoid making the assumption about the higher regularity for the initial data, we make