Nonlinear Analysis: Real World Applications 33 (2017) 226–236

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

A quasistatic viscoplastic contact problem with normal compliance, unilateral constraint, memory term and friction☆

Anna Kulig*

Jagiellonian University in Krakow, Faculty of Mathematics and Computer Science, Chair of Optimization and Control, ul. Lojasiewicza 6, PL-30348 Krakow, Poland

ARTICLE INFO

Article history:
Received 9 April 2016
Received in revised form 15 June 2016
Accepted 16 June 2016

Keywords:
Viscoplasticity
Hemivariational inequality
Normal compliance
Subdifferential
Contact problem
Friction

ABSTRACT

The goal of this paper is to deal with a mathematical model which describes the quasistatic frictional contact between a viscoplastic body and a foundation. The contact is modeled with normal compliance, unilateral constraint and memory term. We present the classical formulation of the problem together with the list of assumptions on the data. Then we derive the variational–hemivariational formulation of the model and we prove its unique weak solvability. The proof is based on a recent abstract result of a class of history-dependent variational–hemivariational inequalities.

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1. Introduction

In this paper we investigate a frictional contact for rate-type viscoplastic materials. We model the materials' behavior with a constitutive law of the form

$$\dot{\sigma}(t) = \mathcal{E}\varepsilon(\dot{u}(t)) + \mathcal{G}(\sigma(t), \varepsilon(u), \kappa(t)) \quad \text{in } \Omega, \tag{1}$$

where u denotes the displacement field, σ the stress tensor, $\varepsilon(u)$ is the linearized strain tensor and κ denotes an internal state variable. Operator $\mathcal{E}$ represents the elastic properties of the material whereas $\mathcal{G}$ and a nonlinear constitutive function which describes its viscoplastic behavior.

The internal state variable κ is a vector-valued function whose evolution is governed by the differential equation

$$\dot{\kappa}(t) = G(\sigma(t), \varepsilon(u(t)), \kappa(t)) \quad \text{in } \Omega, \tag{2}$$

☆ This research was supported by the Marie Curie International Research Staff Exchange Scheme Fellowship within the 7th European Community Framework Programme under Grant Agreement No. 295118 and the National Science Center of Poland under Maestro Advanced Project no. DEC-2012/06/A/ST1/00262.

* Fax: +48 12 664 6673.
E-mail address: anna.1.kulig@uj.edu.pl.

http://dx.doi.org/10.1016/j.nonrwa.2016.06.007
1468-1218/© 2016 Published by Elsevier Ltd.

where G is a nonlinear constitutive function whose values are in $\mathbb{R}^m$ with m being a positive integer. This type of variable was described in [1,2] together with various results, examples and mechanical interpretations in the study of viscoplastic materials of the form (1), (2). Quasistatic contact problems for such materials have been considered in [3,4] where the contact was frictionless and modeled with normal compliance whereas dynamic ones were studied in [5–8].

On the other hand, in papers of [9,10] the contact was modeled with normal compliance and unilateral constraint and in [11] the memory term was added to the contact condition.

Considering the nonmonotone character of the multivalued boundary conditions, a convex analysis approach to the contact problem is not possible. This leads to a mathematical model that involves the Clarke subdifferential of a locally Lipschitz functional. This often leads to hemivariational inequalities. Therefore the contact is modeled with multivalued friction law of the form

$$-\sigma_\tau(t) \in \partial j_\tau\Big(\int_0^t \|u_\tau(s)\|_{\mathbb{R}^d} ds,\, u_\tau(t)\Big) \quad \text{on } \Gamma_3 \tag{3}$$

where ∂j_τ denotes a Clarke subdifferential of a function which is locally Lipschitz and, in general, nonconvex in its second variable.

The notion of hemivariational inequality was introduced by Panagiotopoulos in the middle of 1980s (cf. [12,13]). It is used in variational formulations for certain classes of mechanical problems with the nonconvex and nonsmooth energy functionals. Such formulations allow to achieve the existence results for many engineering problems that involve multivalued and nonmonotone relations. One can find mathematical results on the stationary hemivariational inequalities in [14,15,12].

With respect to the papers mentioned above, the current paper has one novelty. It composes the model involving a contact with normal compliance condition, unilateral constraint and memory term described in [11] together with multivalued friction law of the form (3).

The rest of the paper is structured as follows. In Section 2 we present some preliminary material together with the notation. In Section 3 we describe the model of the contact process. Finally, in Section 4 we present the weak formulation of the problem in the form of a system of two equations and a hemivariational–variational inequality together with its existence result.

2. Preliminaries

In this section we recall the notation and basic definitions needed in the sequel.

First let us recall that a function $h\colon X \to \mathbb{R}$ defined on a Banach space X is called locally Lipschitz, if for every $u \in X$ there exists a neighborhood $\mathcal{N}(u)$ of u such that

$$|h(y) - h(z)| \le K_u \|y - z\|_X \quad \text{for all } u, z \in \mathcal{N},\ K_u > 0.$$

The generalized directional derivative of Clarke of locally Lipschitz function $h\colon X \to \mathbb{R}$ at $x \in X$ in the direction $v \in X$, denoted by $h^0(x; v)$, is defined by (cf. [16])

$$h^0(x; v) = \limsup_{y \to x, \lambda \downarrow 0} \frac{h(y + \lambda v) - h(y)}{\lambda}.$$

The generalized gradient of a function $h\colon X \to \mathbb{R}$ at $x \in X$, denoted by $\partial h(x)$, is a subset of a dual space X^* given by

$$\partial h(x) = \{\zeta \in X^* \,:\, h^0(x; v) \ge \langle \zeta, v \rangle_{X^* \times X} \text{ for all } v \in X\}. \tag{4}$$

The linear space of second order symmetric tensors on $\mathbb{R}^d$ will be denoted by $\mathbb{S}^d$. The inner product and the corresponding norm on $\mathbb{S}^d$ is defined similarly to the inner product on $\mathbb{R}^d$, i.e.

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