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Solitons, breathers and rogue waves for the coupled Fokas–Lenells system via Darboux transformation

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ABSTRACT

In this paper, a vector generalization of the Fokas–Lenells system, which describes for nonlinear pulse propagation in optical fibers by retaining terms up to the next leading asymptotic order, is investigated. Higher-order soliton, breather, and rogue wave solutions of the coupled Fokas–Lenells system are derived via the n-fold Darboux transformation. Meanwhile the dynamic characteristics of those solitary wave solutions have been discussed.

1. Introduction

In [1], Fokas proposed an integrable generalization of the nonlinear Schrödinger (NLS) equation

$$iu_t - \upsilon u_{xt} + \gamma u_{xx} + \sigma |u|^2 (u + i\upsilon u_x) = 0, \quad \sigma = \pm 1, \tag{1.1}$$

by using bi-Hamiltonian method, where γ and υ are nonzero real parameters and $u = u(x,t)$ is a complex-valued function of x and t, and subscripts x and t appended to u denote partial differentiations. Eq. (1.1), known as Fokas–Lenells equation (FL) is a completely integrable equation [2], and reduces to the NLS equation when $\upsilon = 0$. It arises as a model for nonlinear pulse propagation in monomode optical fibers and is the first negative member of the integrable hierarchy associated with the derivative NLS equation in the same way that the Camassa–Holm equation is related to the KdV equation. Furthermore, Lenells and Fokas used the bi-Hamiltonian structure to write down the first few conservation laws of Eq. (1.1) and derived

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its Lax pair, where they solved the initial value problem and analyzed soliton solutions [3]. Recently, many scholars derived explicit formulas for the n-bright and the dark soliton solutions of the FL equation by using several ways, such as the dressing method, Bäcklund transformation, the bilinear method and so on [4–8]. Fan also discussed the long-time asymptotic behavior of the solution of the FL equation by the nonlinear steepest descent method [9]. In addition, He, Xu and Porsezian constructed the breathers and rogue waves of the FL equation by using the Darboux transformation (DT) method [10,11], and revealed that there existed the differences between the FL system and other integrable equations, such as the AKNS system and KN system.

Since the interaction of waves of different frequencies gives rise to vector NLS system, their multi-component generalizations have attracted much attention. The most famous example might be Manakov's system, which is characterized by equal nonlinear interaction between two components. Similarly, another recent integrable generalization of Manakov's system, which is called the coupled Fokas–Lenells (in short, CFL) system [12], is given by

$$\begin{pmatrix} p_1 \\ p_2 \\ r_1 \\ r_2 \end{pmatrix}_t = i \begin{pmatrix} \gamma u_{1,xx} - 2p_1 u_1 v_1 - p_1 u_2 v_2 - p_2 u_1 v_2 \\ \gamma u_{2,xx} - 2p_2 u_2 v_2 - p_2 u_1 v_1 - p_1 u_2 v_1 \\ -\gamma v_{1,xx} + 2r_1 u_1 v_1 + r_1 u_2 v_2 + r_2 u_2 v_2 \\ -\gamma v_{2,xx} + 2r_2 u_2 v_2 + r_2 u_1 v_1 + r_1 u_1 v_2 \end{pmatrix}, \tag{1.2}$$
$$p_k = u_k + i\upsilon u_{k,x}, \qquad r_k = v_k - i\upsilon v_{k,x}, \quad k = 1, 2$$

where γ and υ are nonzero real parameters and $u_k(x,t), v_k(x,t)$ are complex-valued functions. In fact, taking $u_1 = -\tilde{u}, v_1 = \frac{1}{2}\tilde{v}, u_2 = v_2 = 0, p_1 = -\tilde{p}, r_1 = \frac{1}{2}\tilde{r}$, Eqs. (1.2) reduce to

$$\begin{pmatrix} \tilde{p} \\ \tilde{r} \end{pmatrix}_t = i \begin{pmatrix} \gamma \tilde{u}_{xx} + \tilde{p}\tilde{u}\tilde{v} \\ -\gamma \tilde{v}_{xx} - \tilde{r}\tilde{u}\tilde{v} \end{pmatrix}, \tag{1.3}$$

which is just the equation discussed in [3]. Moreover, Eqs. (1.2) can also be written by taking $v_k = \sigma u_k^*, \sigma = \pm 1$ in the following form:

$$iu_{1,t} - \upsilon u_{1,xt} + \gamma u_{1,xx} + \sigma(2|u_1|^2 + |u_2|^2)(u_1 + i\upsilon u_{1,x}) + \sigma u_1 u_2^*(u_2 + i\upsilon u_{2,x}) = 0, \tag{1.4a}$$
$$iu_{2,t} - \upsilon u_{2,xt} + \gamma u_{2,xx} + \sigma(2|u_2|^2 + |u_1|^2)(u_2 + i\upsilon u_{2,x}) + \sigma u_2 u_1^*(u_1 + i\upsilon u_{1,x}) = 0, \tag{1.4b}$$

where asterisk denotes the complex conjugation.

In this paper, motivated by the investigation for the FL system (1.1) and Manakov's system, we consider the following CFL system which can be derived from its original version Eqs. (1.4) by a simple change of variables combined with a gauge transformation [2] ($u_k = e^{ix} q_k, k = 1, 2$) and the condition $\gamma = 2, \upsilon = 1$, $\sigma = -1$,

$$iq_{1,xt} - 2iq_{1,xx} + 4q_{1,x} - (2|q_1|^2 + |q_2|^2)q_{1,x} - q_1 q_2^* q_{2,x} + 2iq_1 = 0, \tag{1.5a}$$
$$iq_{2,xt} - 2iq_{2,xx} + 4q_{2,x} - (2|q_2|^2 + |q_1|^2)q_{2,x} - q_2 q_1^* q_{1,x} + 2iq_2 = 0. \tag{1.5b}$$

The above integrable CFL system has been derived by means of the bi-Hamiltonian method which originated from the idea of Fokas and Fuchssteiner [13]. In particular, a Lax pair and a few conservation laws associated with Eqs. (1.2) have also been obtained explicitly. Another remarkable feature of the CFL system is that it is the first negative flow of the integrable hierarchy of the coupled NLS equation.

Recently, the solitary wave solutions which include bright or dark solitons, breathers, rogue waves and rational solutions for the coupled NLS equation have been widely investigated [14–18]. Likewise, here our aim is to propose a simple method to construct these solitary wave solutions for the CFL system. It is remarked that the DT is a very valid tool to generate soliton solutions of an integrable equation from a seed solution [19]. Indeed, the multiple soliton solutions can be constructed through iterations of the DT.